

IEA
SOLAR R&D

INTERNATIONAL ENERGY AGENCY

solar heating and
cooling programme

task V

**use of existing meteorological information
for solar energy application**

**handbook of methods of
estimating solar radiation**

nov 1984

INTRODUCTION TO THE INTERNATIONAL ENERGY AGENCY AND THE IEA SOLAR HEATING AND COOLING PROGRAMME

The International Energy Agency was formed in November 1974 to establish cooperation among a number of industrialized countries in the vital area of energy policy. It is an autonomous body within the framework of the Organization for Economic Cooperation and Development (OECD). Twenty-one countries are presently members, with the Commission of the European Communities also participating in the work of the IEA under a special arrangement.

One element of the IEA's programme involves cooperation in the research and development of alternative energy resources in order to reduce excessive dependence on oil. A number of new and improved energy technologies which have the potential of making significant contributions to global energy needs were identified for collaborative efforts. The IEA Committee on Energy Research and Development (CRD), supported by a small Secretariat staff, is the focus of IEA RD&D activities. Four Working Parties (in Conservation, Fossil Fuels, Renewable Energy, and Fusion) are charged with identifying new areas for cooperation and advising the CRD on policy matters in their respective technology areas.

Solar Heating and Cooling was one of the technologies selected for joint activities. During 1976-77, specific projects were identified in key areas of this field and a formal Implementing Agreement drawn up. The Agreement covers the obligations and rights of the Participants and outlines the scope of each project or "task" in annexes to the document. There are now eighteen signatories to the Agreement:

Australia
Austria
Belgium
Canada
Denmark
Commission of the
European Communities
Federal Republic of
Germany
Greece
Italy
Japan
Netherlands
New Zealand
Norway
Spain
Sweden
Switzerland
United Kingdom
United States

The overall programme is managed by an Executive Committee, while the management of the individual tasks is the responsibility of Operating Agents. The tasks of the IEA Solar Heating and Cooling Programme, their respective Operating Agents, and current status (ongoing or completed) are as follows:

- Task I Investigation of the Performance of Solar Heating and Cooling Systems – Technical University of Denmark (Completed).
- Task II Coordination of Research and Development on Solar Heating and Cooling – Solar Research Laboratory – GIRIN, Japan (Completed).
- Task III Performance Testing of Solar Collectors – University College, Cardiff, U.K.
- Task IV Development of an Insulation Handbook and Instrument Package – U.S. Department of Energy (Completed).
- Task V Use of Existing Meteorological Information for Solar Energy Application – Swedish Meteorological and Hydrological Institute (Completed).
- Task IV Performance of Solar Heating, Cooling, and Hot Water Systems Using Evacuated Collectors – U.S. Department of Energy (Ongoing).
- Task VII Central Solar Heating Plants with Seasonal Storage – Swedish Council for Building Research (Ongoing).
- Task VIII Passive and Hybrid Solar Low Energy Buildings – U.S. Department of Energy (Ongoing).
- Task IX Solar Radiation and Pyranometry Studies – Canadian Atmospheric Environment Service (Ongoing).
- Task X Materials Research & Testing – Solar Research Laboratory, GIRIN, Japan (Ongoing).

Task V – Use of existing meteorological information for solar energy application

The objectives of Task V are to improve the availability of existing solar radiation and related meteorological data and to support the collection and presentation of such data in an effective manner for the solar energy community.

The project is comprised of the following subtasks:

- A Compilation of Sources of Solar Radiation and Relevant Meteorological Data
- B Preparation of a Handbook on Estimation Methods
- C Recommendations Concerning Meteorological Stations
- D Preparation of a Uniform Format for Presentation of Data.

The following countries are participants in this task: Austria, Belgium, Canada, Denmark, Germany, Italy, the Netherlands, Spain, Sweden, Switzerland, United Kingdom, USA, and the Commission of European Communities.

This report documents work carried out under subtask B of this task.

handbook of methods of estimating solar radiation

The Swedish Meteorological and
Hydrological Institute, SMHI,
Norrköping, Sweden

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Distribution: unrestricted

This report is part of work within the IEA Solar Heating and Cooling Programme,
Task V: Use of Existing Meteorological Information for Solar Energy Appli-
cation

Subtask B: Preparation of a Handbook on Estimation Methods

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TASK V

SUBTASK B

HANDBOOK ON METHODS OF ESTIMATING

SOLAR RADIATION

PREFACE

International Energy Agency

The International Energy Agency (IEA), an autonomous body within the Organization for Economic Cooperation and Development (OECD), established in 1975 an International Energy Programme by which its twenty-one member countries cooperate in the area of energy policy. Joint research, development and demonstration activities in new and improved energy technologies comprise one element of the IEA programme. The objective of these collaborative activities is to further the development of various energy technologies through pooling of resources, ideas, and expertise.

Solar Heating and Cooling Programme

Solar Heating and Cooling was among the technologies selected for collaboration, and in 1977 an "Implementing Agreement for a Programme to Develop and Test Solar Heating and Cooling Systems" was developed. During the course of the programme, a total of nine annexes to the Implementing Agreement have been developed, each annex outlining a cooperative task in a particular aspect of solar heating and cooling. Each task is led by an Operating Agent, with an Executive Committee (consisting of representatives from each signatory to the Implementing Agreement) responsible for management of the overall programme.

The tasks of the IEA Solar Heating and Cooling Programme and their respective Operating Agents are:

- I. Investigation of the Performance of Solar Heating and Cooling Systems - Technical University of Denmark
- II. Coordination of R & D on Solar Heating and Cooling Components - Agency of Industrial Science and Technology, Japan
- III. Performance Testing of Solar Collectors - Kernforschungsanlage Jülich, Federal Republic of Germany

IV.* Development of an Insolation Handbook and Instrumentation Package - United States Department of Energy

V. Use of Existing Meteorological Information for Solar Energy Application - Swedish Meteorological and Hydrological Institute

VI. Performance of Solar Heating, Cooling and Hot Water Systems using Evacuated Collectors - United States Department of Energy

VII. Central Solar Heating Plants with Seasonal Storage - Swedish Council for Building Research

VIII. Passive and Hybrid Solar Low Energy Buildings - United States Department of Energy

IX. Solar Radiation and Pyranometry Studies - Atmospheric Environment Service, Canada

*) Task IV was completed in 1979.

Task Description

The objectives of Task V are to increase access to needed solar radiation and other meteorological data, facilitate the calculation of solar radiation required by the solar engineers and designers, and develop a uniform format for presentation of data.

In order to achieve these objectives, the following subtasks were undertaken:

- A. Sources of Solar Radiation Data and Relevant Meteorological Data - Compilation of a catalogue on data sources worldwide, their measurement programs, and types of format of data available.
- B. Handbook of Methods of Estimating Solar Radiation - An overview of existing methods of estimating solar radiation on horizontal and inclined surfaces.

C. Recommendations Concerning Meteorological Networks - Development of recommendations for a minimum meteorological network to obtain necessary measurements for solar energy applications.

D. Format for Presentation of Data - Compilation of effective formats for presentation of meteorological data and recommendations for uniform international formats for the solar energy community.

Task Participants

Austria	The Netherlands
Belgium	Sweden
Canada	Switzerland
Denmark	United Kingdom
Federal Republic of Germany	United States Commission of the
Italy	European Communities

Handbook of Methods of Estimating Solar Radiation

In its original concept this Task V report, consisting of a survey of available methods of estimating solar radiation on horizontal and inclined surfaces, was to include systematic information on the accuracy of each of the methods. This information, it was thought, would permit a user to choose the best available estimating model for his application when considering available data and demands for accuracy.

During the course of the work it became clear that suitable information on the model accuracies was not available simply because there had been so few adequate validations of models. Furthermore, the wide variety of techniques that had been used to describe model performance made comparison difficult. Therefore, to meet the original objective would necessitate initially that methods be classified, that models be catalogued uniformly, that statistical performance criteria be reviewed and that data sets suitable for validation be compiled. Later, a number of selected models would have to be validated. Altogether, the amount and duration of work would have been much greater than originally anticipated for the present task.

In view of the otherwise large extension to Task V, it was decided not to undertake model validation. This report, which is indeed a current overview of estimation methods, deals precisely with those activities which are listed above as prerequisites for model validation. The report contains less information on model accuracy than was hoped for, but it will serve as a basis for future work on the evaluation of models.

The testing of models has been left to a new meteorological annex of the IEA Solar Heating and Cooling Programme named "Task IX, Solar Radiation and Pyranometry Studies". In Task IX, models will be validated according to objective and uniform criteria with real data collected in climatic regions different from those on which the models were developed. Thus, the data sets which have been compiled by Task V will be supplemented by Task IX so as to extend over longer time periods and to cover more climate types.

The Handbook of Methods of Estimating Solar Radiation begins with the classification of estimating models. This is the subject of chapter 1 which refers extensively to the two contributions, one by P Bener and J A Davies, which are placed as appendices. Chapter 2 contains a catalogue of estimating methods in which selected models are described in a uniform manner. Various statistical techniques that have been used to evaluate performance of estimation models are reviewed in chapter 3. Chapter 4 describes available measured data sets of solar radiation and relevant meteorological observations of potential use for model validation.

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CHAPTER 1

CLASSIFICATION OF METHODS FOR COMPUTING

THE COMPONENTS OF SOLAR IRRADIANCE

by

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Introduction

Over the years there have been several attempts to develop methods of computing the solar irradiance and its direct beam and diffuse components on a horizontal surface and on sloping surfaces at any location. The requirements for such procedures arise from the fact that the radiation networks established in most countries are too sparse to provide sufficient data in a spatial and temporal sense to define the climatological potential for solar energy utilization and other related research and applications. Therefore, the need exists for methods to estimate solar irradiance at locations where there are no measurements and at locations where there are gaps in the measurement record.

Recently two independent studies (Bener 1980, Davies 1981) have been conducted to produce a survey of the various methods or models for estimating solar irradiance. The two studies are included as appendices (Appendix B and C) to this Handbook and this chapter will just mention where similarities exist between the Bener and Davies reports. One of the objectives of the Davies report was to provide a classification and discussion of models which compute solar irradiance and its direct beam and diffuse components on horizontal surfaces. Bener, while discussing models which compute solar irradiance on horizontal surfaces, also examines models which compute global, direct and sky radiation on sloping surfaces. Since Davies does not discuss models for estimating solar irradiance on inclined surfaces, only models for computing solar irradiance on horizontal surfaces will be mentioned here. For an examination of solar radiation models for sloping surfaces, the reader is referred to chapters 7, 8, 9 and 10 of Bener's report. Appendix A provides a list of symbols which have similar definitions in both reports. Each report contains a more comprehensive list of symbols.

Radiation Transfer and Classification of Models

The solar irradiance received at the

ground is a function of the irradiance received at the top of the atmosphere and scattering and absorption by atmospheric constituents. Both Bener (sec. 3.1) and Davies (sec. 1.2) discuss the transfer process in its most general form. The problems in attempting to achieve an exact solution are examined - in particular the difficulty in obtaining a rigorous solution to the problem of multiple scattering. As noted by the authors, the most exact solutions are only obtained by means of elaborate and costly computer programs and generally they have to be applied to model atmospheres, and are not practical for calculating irradiances routinely.

With the difficulties involved in obtaining formal solutions, approximations using varying degrees of empiricism have been developed. Some of the approximations have maintained close links with the radiative transfer equation while with others, the link is subtle. Davies has presented a classification of models which encompasses most types of models and emphasizes interrelationships between models. Models were classified into five groups, four of which have some physical bases and a fifth which is entirely empirical. The five groups are:

1. Cloudless sky models
2. Cloud layer based models
3. Total cloud based models
4. Sunshine based models
5. Liu and Jordan type models

Each group is reviewed individually in the Davies report. Table 1 indicates the section (in the Davies report) where each classification is reviewed and the sections in the Bener report where models related to Davies' classification are reviewed by that author.

Both reports provide an extensive list of references which together provide a comprehensive survey of the various models presently being used to calculate solar irradiance and its direct and diffuse components on horizontal surfaces.

Table 1. Classification of solar radiation models by Davies and where they are reviewed in Davies and Benerreports

<u>Bener</u> (section)	<u>Classification of Models</u>	<u>Davies</u> (section)
1.2, 1.3	1. Cloudless sky models (a) Direct Beam	1.4.1(a)
3.2	(b) Diffuse	1.4.1(b)
4.0	(c) Global	1.4.1(c)
-	2. Cloud Layer based models	1.4.2
-	3. Total Cloud based models	1.4.3
5.1, 5.2	4. Sunshine based models	1.4.4
6.1	5. Liu and Jordan type models	1.4.5

CHAPTER 2

CATALOGUE OF ESTIMATION METHODS

by

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2.1 Introduction

This chapter gives brief descriptions of some existing methods of estimating, in the absence of direct measurements of the quantity required, the solar irradiation and irradiance on a plane receiver at the surface of the earth. In particular it is often necessary to be able to estimate the irradiance on a slope of arbitrary orientation. To do this successfully, it is necessary to estimate the individual contributions of the three components of the solar radiation, the direct beam contained in a narrow cone centred on the direction of the sun, the diffuse component coming from the remainder of the sky and the component reflected from the ground. In certain circumstances, the first and third component may be zero. Thus, with cloud between the sun and the receiving surface, the direct beam component is zero, and there is no ground reflected component on a horizontal surface.

Sections 2.3-2.15 of this chapter describe the methods used by selected groups of workers using a common format. These descriptions are based extensively on a literature survey carried out by B.R. May of the United Kingdom Meteorological Office.

Section 2.2 of this chapter is a brief commentary on the succeeding sections, indicating the general scope and complexity of the methods described.

2.2 Brief Comparison of Methods

The most well-developed and well-tested method of estimating the solar irradiance and irradiation on any plane is probably that described in Section 3, which has been developed at the Department of Building Science at Sheffield University under the leadership of Professor J.K. Page.

Once access to the suite of computer programmes is available, it is very easy to use, even by those unfamiliar with computer operations. It has been extensively tested on a wide range of European data, but it has also been designed to be of worldwide use.

Section 2.4 (R. Dogniaux), Section 2.5 (S.A. Klein) and Section 2.6 (R.C. Temps and K.L. Coulson) are somewhat similar attempts to derive the solar irradiances on any arbitrary plane, but these have not, in general, been based on as much data or subjected to such extensive comparisons as the method described in Section 2.3. They may, however, prove to be of value for the areas in which their data sources are located.

Section 2.15 (McKay and Associates) is another comprehensive formulation from which, given hourly values of standard meteorological data (cloud, humidity, air temperature, pressure), hourly radiation data can be computed. It has been extensively tested on Canadian data.

Section 2.9 (J.W. Buglar) also describes a useful general method but it was developed using Australian data and, as yet, has had only limited validation.

Sections 2.7 (K.Ya. Kondratyev) and 2.10 (N. Robinson and W. Schüepp) are of more theoretical nature and are probably of most interest to meteorologists, and others wishing to know more of the general basis.

Section 2.8 (B.Y.H. Liu and R.C. Jordan) is a description of some of the earlier work in this area and only deals with daily totals of solar irradiation. Although the method can probably be extended to any arbitrary plane, the study reported was restricted to tilted surfaces facing South.

Section 2.11 (A. De Vos and G. De May) is a study confirmed to yearly totals of solar irradiance, and is mainly applied to the situation in Belgium.

Section 2.12-2.14 are also of more limited application. Section 2.12 (K.J.A. Revfeim) deals only with the direct component of the solar radiation; Section 2.13 (T. Kusuda and K. Ishii) deals with hourly irradiances but is confined to horizontal and vertical surfaces in the United States and Canada; Section 2.14 (L.W. Swift and K.R. Knoerr) deals with daily totals of solar irradiation on mountain slopes.

2.3 EEC Solar Energy Programme Project F Action 3.2 (Predetermination of irradiation on inclined surfaces for different European centres).

References:

Final Report prepared by Department of Building Science, University of Sheffield, October 1979. (Volume I of this report describes the mathematical basis of this scheme and Volume II describes the detailed studies for specific European sites).

Authors:

J.K. Page, G.G. Rodgers, C.G. Souster, S.A. Le Sage. Enquiries to be addressed to Professor J.K. Page, Department of Building Science, University of Sheffield, Sheffield, S10 2TN, England.

Abstract:

A research group in the Department of Building Science, University of Sheffield has carried out a systematic programme of study for the purpose of providing an improved design methodology for solar houses based on interactive computing methods. The methodology developed covers all latitudes in the world and is designed to be both flexible and very easy to use by those who are not skilled in computer techniques. A modular approach has been adopted towards the production of the computer programmes.

The final aim is to combine the models into a thermal energy balance model which will allow the thermal performance of a solar house design to be examined, systematically, very quickly.

Description:

Modules described are:

- (i) Some which compute direct and diffuse irradiances from a clear sky on horizontal or inclined surfaces or on surfaces which follow the sun.
- (ii) One which computes mean monthly values of mean hourly global, diffuse and direct irradiances on horizontal surfaces from sunshine data. The methodology here is used as the foundation of a method coping with inclined surfaces, described more fully in ref. 1.

The clear-sky model is not described in detail. It follows closely the theory of Unsworth (ref 2), being based on a turbidity coefficient, τ_a , which relates the measured direct normal irradiance, G_{bn} , to the clean atmosphere direct normal irradiance, G_{bn}^* , by the formula:

$$G_{bn} = G_{bn}^* \exp(-\tau_a m)$$

where m = optical air mass.

G_{bn}^* allows for known amounts of scattering and absorbing gases and is corrected to the mean solar distance.

If values of m , τ_a and precipitable water content of the atmosphere are known, then the direct irradiance at normal incidence can be determined from the above equation.

Diffuse radiation model for clear skies: Using Parmelee's data (ref 3) a relationship was found to exist between diffuse and direct irradiances (G_{dh} and G_{bh}), viz:

$$G_{dh} = a_0 - a_1 G_{bh} \quad Wm^{-2}$$

a_0 and a_1 are constants for a particular solar altitude.

This equation is incorporated into the Sheffield clear sky programmes. Thus, knowing the solar altitude, it is possible to derive the direct irradiance on a horizontal surface, G_{bh} , from the Monteith and Unsworth turbidity and estimate the corresponding associated diffuse clear sky irradiance, G_{dh} , on a horizontal surface.

The average-day model:

The direct component: The average day model is based on mean monthly values of horizontal hourly mean global and diffuse irradiances (\bar{G}_h and \bar{G}_{dh}) at Kew over the period 1965-1975 (excluding 1973). An estimate of the mean horizontal direct irradiance (\bar{G}_{bh}) at each hour in a particular month was made by subtracting \bar{G}_{dh} from \bar{G}_h . Because each hourly value of \bar{G}_{bh} is obtained by averaging over the whole hour when in fact bright sunshine was on average only recorded for a fraction, f , of that hour a more realistic value for the instantaneous direct

horizontal surface irradiance during the fraction of the hour for which the sun was shining is \bar{G}_{bh}/f . This figure allows a pseudo-turbidity, τ'_a , to be determined, according to the Unsworth and Monteith definition as

$$\tau'_a = -\frac{1}{m} \left[\frac{\bar{G}_{bh}}{f} \times \frac{1}{G_{bh}^* \cdot \sin \alpha} \right]$$

α = solar altitude

In many situations only the mean daily total of bright sunshine for particular month, \bar{n}_o , is known. In these cases, an average value of f is derived and applied to each hour of the day. This average value of f is obtained as the ratio of \bar{n}_o to the maximum possible number of hours of bright sunshine, n_r , which would be recorded on a totally clear day in the middle of each month. To obtain n_r it is assumed that "bright sunshine" corresponds to a value of G_{bh} greater than 200 Wm^{-2} and the clear sky programme is run using a fixed reference turbidity of 0.2 in order to determine the number of hours for which the predicted G_{bh} is greater than 200 Wm^{-2} . Using this monthly fraction, f , the diurnal variation in the pseudo-turbidity at Kew has been determined for each month of the year. These diurnal curves have been fitted by an appropriate cosine function whose arguments vary with month and are stored in the average day programme.

Hourly value of \bar{G}_{bh} obtained from the pseudo-turbidities are finally integrated over the day to give the mean daily total direct irradiation on the horizontal, \bar{H}_{bh} .

The diffuse component: The model uses a relationship between mean monthly hourly diffuse irradiances and solar elevation:

$$G_{dh} = 2 + 4.804 \alpha \quad \text{Wm}^{-2} \quad (i)$$

where α is in degrees.

This relationship was found to exist using data for Kew, 1965-75 (excluding 1973).

When this relationship is applied to other sites, allowance is made for the difference in sunshine hours recorded at that site with respect to Kew, as follows:

Combining the regression equations

$$\frac{\bar{H}_{dh}}{\bar{H}_{oh}} = a + b \frac{\bar{n}}{N_o} \quad \text{and} \quad \frac{\bar{H}_{dh}}{\bar{H}_h} = c + d \frac{\bar{H}_h}{\bar{H}_{oh}}$$

(shown by Page to represent mean monthly irradiances -ref 4) gives:

$$\frac{\bar{H}_{dh}}{\bar{H}_{oh}} = e + f \frac{\bar{n}}{N_o} + g \left[\frac{\bar{n}}{N_o} \right]^2$$

where \bar{H}_{dh} is the mean monthly daily diffuse irradiation on a horizontal plane.

\bar{H}_{oh} is the mean monthly daily irradiation on a horizontal plane in the absence of any atmosphere.

\bar{H}_h is the mean monthly daily global irradiation on at horizontal plane.

\bar{n} is the monthly mean daily hours of bright sunshine.

N_o is the mean daily number of hours of daylight in a particular month.

e, f, g are constants derived from a, b, c and d .

Using Kew data and the method of least squares, the following fit to the data was obtained:

$$\frac{\bar{H}_{dh}}{\bar{H}_{oh}} = 0.1534 + 0.2891 \frac{\bar{n}}{N_o} + 0.2442 \left[\frac{\bar{n}}{N_o} \right]^2$$

Using this equation, daily diffuse horizontal irradiances are deduced from daily sunshine hours for the site of interest. The mean hourly diffuse irradiances are then obtained by multiplying the values given by equation (i) by the ratio of the daily diffuse irradiation at the site to the daily diffuse irradiation at Kew, viz:

$$\bar{G}_{dh} \text{ at site} = (2 + 4.804\alpha) \cdot \frac{\bar{H}_{dh} \text{ at site}}{\bar{H}_{dh} \text{ at Kew}}$$

In ref 1, Page explains how the Kew horizontal surface model is corrected to allow for local climatical variations and how the horizontal direct and diffuse components are converted into the corresponding components on inclined surfaces.

Mean daily diffuse irradiation (average day model):

The equation $\frac{\bar{H}_{dh}}{\bar{H}_h} = c + d \frac{\bar{H}}{\bar{H}_{oh}}$

(Page, ref 4) is rearranged and used to show that the maximum possible value of \bar{H}_{dh} is

$$\frac{\bar{H}_h}{\bar{H}_{oh}} = -\frac{c^2}{4d}$$

\bar{H}_h , \bar{H}_{dh} and \bar{H}_{oh} are respectively the mean monthly values of daily global, diffuse and 'in absence of any atmosphere', irradiances on a horizontal plane.

The values of $\frac{c^2}{4d}$ at various sites are expressed as ratios of the value of $\frac{c^2}{4d}$ at Kew. This ratio is then used as a diffuse multiplier to correct the mean diffuse irradiance figures on the computer based on Kew data to match actual observations at other stations more accurately.

Average-day models for inclined surfaces: The procedure uses mean monthly sunshine duration as its starting material and the method by which direct and diffuse irradiances on horizontal surfaces are derived from these data are described above. The direct component on inclined surfaces is computed by trigonometrical relationships.

Diffuse radiation is treated by separating it into 2 components: i) that arriving from the clear portion of the sky and ii) that arriving from the overcast portion of the sky.

(i) The clear sky diffuse component is itself considered as comprising two parts, namely a uniform background irradiance

and an area of high irradiance in the vicinity of the sun which is proportional to the direct normal incidence irradiance.

(ii) The overcast sky diffuse component is computed according to the Moon and Spencer distribution (see ref 6). This is defined by the equation

$$L_\theta = L_z \left(\frac{1}{3} + \frac{2}{3} \sin \theta \right)$$

which relates the radiance of an overcast sky, L_θ , at angle θ from the horizontal plane, to the zenith radiance, L_z .

Data requirements:

Clear sky model: Unsworth and Monteith's turbidity coefficient (τ_a), atmospheric precipitable water.

Average day model: Mean monthly values of daily or hourly sunshine duration. If no observed values are available estimates can be made.

Unsworth and Monteith (ref 2) give some information on the variation in τ_a over the UK in summer months.

The authors have also used their computer programmes to derive statistical values of τ_a at various UK sites for different classes of radiation day.

Recommended values of τ_a and of precipitable water in the UK are given for different classes of radiation day in a Department of Building Science internal note (ref 5).

Accuracy:

The results of using this system have been extensively checked on behalf of the EEC by using solar radiation data from about 22 stations in Europe ranging from Bergen (latitude $60^\circ 24'N$) to Odeillo (latitude $42^\circ 29'N$) (2). In particular, a comparison has been made between predicted and observed data on slopes of various tilts at some 8 stations. The mean differences between observed and predicted values were mostly less than 10%

(many were less than 5%), except when the irradiances themselves were very low, when higher percentage differences were occasionally found. Some results are quoted in chapter 3 appendix E.

Computing:

The system is based on an electronic computer, using programmes written in Fortran, and needs only a modest storage capacity (64 K bytes are ample). The input/output can either be in the batch mode or, interactively, using a teletyp or VDU terminal. Printed output can be in graphical form as well, if suitable peripheral devices are available. Reference should be made to the Department of Building Science (Professor J.K. Page) University of Sheffield, for further details.

Further references:

- 1) J.K. Page: Geographical variations in the climatic factors influencing solar buildings design, Proc NELP/UNESCO Conf, London, Paper 1/2 (1977).
- 2) M.H. Unsworth, J.L. Monteith: Aerosol and Solar Radiation in Britain, Quart J R Met Soc 98, pp 778-797, (1972).
- 3) G.V. Parmelee: Irradiation on vertical and horizontal surfaces by diffuse solar radiation from cloudless skies, Trans Am Soc Heat Vent Engrs 60, pp 341-356, (1960).
- 4) J.K. Page: The estimation of monthly mean values of daily short wave irradiation on vertical and inclined surfaces from sunshine records for latitudes 60°N to 40°S. Internal Research Report No BS 32, Department of Building Science, University of Sheffield (1976)
- 5) C.G. Souster, J.K. Page, G.G. Rodgers: A guide to appropriate values of atmospheric turbidity and precipitable water content for different months and different classes of radiation day in the UK, Internal Note, Departement of Building Science, University of Sheffield, (1976).
- 6) R.G. Hopkinson et al: Daylighting, Chapter 2, pp 29-58, (1968), Heinemann, London.

2.4 Computer Procedure for Accurate Calculation of Radiation Data Related to Solar Energy Utilization.

Reference:

WMO, WMO No 477, pp 191-197 (1977).

Author:

R. Dogniaux.

Abstract:

The paper presents a set of mathematical expressions which directly yield the quantities of direct, diffuse, global and reflected components of solar irradiance and illuminance for the conditions of clear and overcast sky. The method lends itself to programming for computer calculation. It takes account of latitude, date, time, atmospheric turbidity and orientation and inclination of surface. By applying weighting factors (associated with the duration of sunshine) to the extremes of clear sky and overcast sky it is possible to derive standard radiation data for any typical reference period longer than 10 days. Predicted data are compared with measured data from various sites in Belgium, France, Budapest and Oran.

NB: Tables of hourly and daily irradiances on differently orientated planes for latitudes between 0° and 60° and various turbidity factors have been prepared and published in ref 1.

Description:

Direct component: - is computed from the solar constant E_0 (J) using

$$E = E_0 (J) e^{-amT} \cos j$$

where the nomenclature is as follows:

J = the day number in a yearly sequence

a = total extinction coefficient

m = reduced air mass (allowing for pressure and altitude)

T = turbidity factor

j = angle of incidence of direct beam on the receiving surface

(The derivation of these variables will be described under "Data processing" and "Computing").

Diffuse component: The author states the formula

$$D = 0.0076154 \left\{ \begin{array}{l} 87.5 \\ \theta_i = 2.5 \end{array} \right\} \left\{ \begin{array}{l} 357.5 \\ A_i = 2.5 \end{array} \right\} L_i(\theta, A) \cdot \cos j_i \cos \theta_i$$

(derivation not given)

where $L_i(\theta, A)$ is the luminance distribution of the sky and θ, A are respectively the altitude and azimuth angles of a spherical zone of sky.

A different expression is used for $L_i(\theta, A)$ according to whether the sky is cloudy or overcast. For a clear sky:

$$L_i(\theta, A) = L^0(h, T) \cdot \frac{(0.910 + 10e^{-3} + 0.45 \cos^2 j)(1 - e^{-0.32 \operatorname{cosec} \theta})}{0.27385 (0.91 + 10e^{-3(90^\circ - h)} + 0.45 \sin^2 h)}$$

$$\text{with } L^0(h, T) = (0.0023 h^2 + 0.0200 h + 1.3513)T$$

$$\text{and } j = \arccos [\sin \theta \sin h + \cos \theta \cos h \cdot \cos (A - A_0)]$$

h = solar altitude in degrees

For a cloudy sky,

$$L_i(\theta) = \frac{1}{3} L_c^0 (1 + 2 \sin \theta)$$

$$\text{with } L_c^0 = 18.5 \sin h$$

Reflected component: -approximated by the expression:

$$R = 0.5 \alpha (E + D)_H (1 - \cos p)$$

where α = albedo of ground

$(E+D)_H$ = global radiation on a horizontal surface

and p = slope of the receiving surface

Global radiation: See under heading "Computing".

Hourly and daily irradiances are computed by integrating irradiances at half-hourly intervals, using a method of Lagrangian interpolation of the parabolic function representing the hourly variations of irradiances.

The yearly variation of day - totals of radiation for each component is given by the following Fourier series with 7 terms:

$$Y = A + B \cos wJ + C \cos 2 wJ + D \cos 3 wJ + E \sin wJ + F \sin 2 wJ + G \sin 3 wJ.$$

where A, B, \dots, G depend on the latitude of the station and on the turbidity factor

$$\text{and } w = \frac{2\pi}{366}$$

Daily irradiation on inclined surfaces for mean conditions of sunshine:

Let the diffuse, direct and global irradiances be E_o, D_o and G_o for clear skies, and E_s, D_s and G_s for mean conditions over a period of at least 10 days. Then if $\frac{S}{S_o}$ is the ratio of the effective to the maximum theoretical possible sunshine duration over the mean period, an Ångström-type equation can be used to estimate the corresponding global irradiation:

$$G_s = G_o \left(a \frac{S}{S_o} + b \right) = (E_o + D_o) \left(a \frac{S}{S_o} + b \right)$$

where a and b are local parameters, evaluated from measurements of global radiation and sunshine duration on a horizontal surface.

For a long reference period, say 15 years, it is assumed that the sunshine duration is randomly distributed over all hours. Then the direct and diffuse irradiances can be computed for shorter time intervals, say 1 hour, using the same value of $\frac{S}{S_o}$ as is used for daily values:

$$E_s = E_o \left(\frac{S}{S_o} \right)^n$$

$$\text{and } D_s = G_o \left(a \frac{S}{S_o} + b \right) - E_s \left(\frac{S}{S_o} \right)^n$$

where n is a local parameter obtainable from the Meteorological services.

Data requirements:

Latitude, longitude, altitude of site, atmospheric pressure; also values of turbidity coefficient (β) and precipitable water content of the atmosphere (W) to enable computation of the turbidity factor T. For computation of irradiances for mean conditions of cloudiness, daily sunshine duration is required, along with daily totals of global radiation on a horizontal surface (for evaluation of regression constants in the Ångström equation).

The turbidity factor T takes account of the optical air mass, the water content of the atmosphere and the turbidity coefficient β .

$$T = \left[\frac{h + 85}{39.5e^{-W} + 47.4} + 0.1 \right] + (16 + 0.22 W)\beta$$

The author suggests values of W according to climatic type and values of β according to whether the site is rural, urban or industrial (to be used in the absence of more specific data). The precipitable water in cm can also be deduced from the water vapour pressure (e) in mb at ground level, using Hann's formula:

$$W = 0.25 e$$

Accuracy:

The method for mean conditions of cloudiness has been tested for various locations. A typical set of comparisons with measured data are quoted in the paper (for mean values over a 15 year period for a reference day based on 10 days at Uccle, Belgium), as follows:

$$\text{global radiation: } 0.95 < \frac{G_s}{G_o} < 1.03$$

$$\text{direct radiation: } 0.94 < \frac{E_s}{E_o} < 1.08$$

$$\text{sky radiation : } 0.93 < \frac{D_s}{D_o} < 1.02$$

Computing:

The method is designed with computer programming in mind. Though a detailed pro-

gram is not listed and a language not specified the equations for computation of the astronomical and geographical variables needed in the method are given and are listed below:

Solar constant

$$E(J) = 1353 + 45.326 \cos wJ + 0.88018 \cos 2wJ - 0.00461 \cos 3wJ + 1.8037 \sin wJ + 0.09746 \sin 2wJ + 0.18412 \sin 3wJ$$

$$\text{where } w = \frac{2\pi}{366}$$

and J = day number

Solar declination

$$\delta = 0.33281 - 22.984 \cos wJ - 0.34990 \cos 2wJ - 0.13980 \cos 3wJ + 3.7872 \sin wJ + 0.03205 \sin 2wJ + 0.07187 \sin 3wJ$$

Equation of time

$$ET = 0.0072 \cos wJ - 0.0528 \cos 2wJ - 0.0012 \cos 3wJ - 0.1229 \sin wJ - 0.1565 \sin 2wJ - 0.0041 \sin 3wJ$$

Optical air mass

$$m_h = \frac{1}{\sin h + 0.15 (h + 3.885)^{-1.253}}$$

Solar elevation, h, given by

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cdot \cos (TST-12)$$

where ϕ = latitude
and TST = true solar time

Reduced air mass

$$m = m_h \frac{p}{1000} = m_h (1-0.1H)$$

where p = pressure in mb
and H = altitude in km

Total extinction coefficient

$$\bar{a} = 1.4899 - 2.1099 \cos h + 0.6322 \cos 2h + 0.0252 \cos 3h - 1.0022 \sin h + 1.0077 \sin 2h + 0.2606 \sin 3h$$

Further references:

1) Dogniaux, R.: "Variations géographiques et climatiques des expositions énergétiques sur des surfaces réceptrices horizontales et verticales". Int Roy Met de Belgique. Misc. Série B no 38 (1975).

2.5 Calculation of Monthly Average Insolation on Tilted Surfaces.

Reference:

Solar Energy, Oxford, 19, pp 325 - 329, (No 3, 1977).

Author:

S.A. Klein, Solar Energy Laboratory, University of Wisconsin - Madison, Madison, WI 53706, USA.

Abstract:

The method developed by Liu and Jordan (1) for estimating average daily radiation for each calendar month on surfaces facing directly towards the equator is verified with experimental measurements and extended to allow calculation of monthly average radiation on surfaces of a wide range of orientations. Liu and Jordan's method is also compared with that of J.K. Page.

Description:

Refer to Liu and Jordan (1). The extension to surfaces of orientation other than South involves the integration of extra-terrestrial radiation on the surfaces for the period during which the sun is both above the horizon and in front of the surface. This is divided by the mean daily extra-terrestrial radiation on a horizontal surface. A corresponding expression for R_p is given, from which R can be calculated.

Data requirements:

Monthly mean daily \bar{H} .

Accuracy:

No figures quoted for the extension method. Results from Liu and Jordan and Page's methods are compared with measured values, though no figure for the accuracy so achieved is quoted. Note: This comparison table has subsequently been amended: see Solar Energy 20, pp 441, No 5, 1978.

Computing:

Not mentioned.

Further references:

1) B.Y.H. Liu and R.C. Jordan: Daily insolation on surfaces tilted toward the equator: ASHRAE, Vol 3, pp 53 - 59, No 10, Oct 1961

2.6 Solar Radiation Incident Upon Slopes of Different Orientations.

Reference:

Solar Energy, Oxford, 19, pp 179 - 184 (1977).

Author:

R.C. Temps and K.L. Coulson, Department of Land, Air and Water Resources, University of California, Davis, CA 95616, USA.

Abstract:

By means of a pyranometer orientated at various tilt angles and azimuth angles with respect to the sun, measurements of the direct and diffuse solar flux incident on slopes of various orientations have been made for clear sky conditions. The resulting data have been used to test the validity of the solar flux model of Robinson (1) and to introduce certain modifications of that model to achieve better agreement of computations with measurements. It is concluded that the modified model yields significantly better results than the original for slopes orientated in the azimuth of the sun, but results are comparable for other azimuthal orientations of the slope. The accuracy attainable by this method is considered sufficient for many practical applications.

Description:

The basis of Robinson's model is (for cloudless skies) set out on page 113. For the purposes of this study the Ångström turbidity coefficient was actually measured by means of a pyrliometer. The precipitable water content of the atmosphere was estimated using the approximation developed by Hann (2), viz:

$$W = 0.25 e$$

where e is the water vapour pressure measured (in millibars) at the surface. W is measured in centimeters.

Modification of Robinson's method by Temps and Coulson is in the form of empirical corrections to take account of three regions of anisotropy in the diffuse radiation field, viz:

- (i) their observation that the intensity of diffuse sky radiation is 40% greater near the horizon than at the zenith, and that the gradient is strongest at low elevation angles.
- (ii) brightening of the sky in the vicinity of the sun.
- (iii) anisotropy of ground reflection on the basis of reflectance measurements made by Coulson et al (3) for grass turf.

The expressions for D'_{E_s} and D'_r (equations (iv) and (v) of Robinson's method, page 113) then become: -

$$D'_{E_s} = [k (I'_w - I') \sin h \cos^2 \frac{E}{2}]$$

- $[1 + \sin^3 \frac{E}{2}]$ correction (i)
- $[(1 + \cos^2 i) \sin^3 Z]$ correction (ii)

Z = solar zenith angle = $(90^\circ - h)$ and

$$D'_r = (I'_H - D') R \cdot (1 - \cos^2 \frac{E}{2})$$

- $(1 + \sin^2 \frac{Z}{2}) \cdot (| \cos A |)$ correction (iii)

Data requirements:

Solar elevation and azimuth, atmospheric pressure, water vapour pressure at the surface, ground albedo, Ångström's turbidity coefficient.

Accuracy:

No figures are quoted though graphs are presented which compared results from the two models with measured data.

It is stated that the applicability of the methods to other geographical locations and atmospheric conditions requires further study, but that the method should yield useful approximations for clear sky conditions at any location although empirically derived constants (such as precipitable water) may require some adjustment.

Computing:

Not mentioned.

Further references:

- 1) Robinson, N.: Solar Radiation, Elsevier Publishing Co, London 1966).
- 2) Hann J.: Lehrbuch der Meteorologie, Tauchnitz, Leipzig, p 225 (1901).

2.7 Radiation Regime of Inclined Surfaces.

Reference:

WMO Tech Note No 152 (WMO No 467, 1977).

Author:

K.Ya. Kondratyev, Rapporteur on Atmospheric Radiation of the WMO Commission for Atmospheric Sciences (CAS).

Abstract:

This 82 page document is intended to help meteorologists engaged in atmospheric research, as well as solar energy engineers, to understand the relevant meteorological implications in the planning and development of solar energy utilization. The four chapters deal with the receipt of direct, diffuse and global radiation, and with radiation balance on inclined surfaces.

Description:

Direct Component: A mathematical treatment is presented, derived from material presented in ref 1, for calculation of the direct radiation incident on an inclined plane, given the direct irradiance at normal incidence to the sun.

Let S_m , S_H , S_V and S_S be the direct solar irradiances on surfaces orientated, respectively, normal to the solar beam, horizontally, vertically, and in an arbitrarily inclined plane. It is shown that

$$S_s = S_m [A_1 + B_1 \cos \Omega + C_1 \sin \Omega] \quad (i)$$

where Ω = hour angle, and

$$A_1 = \cos \alpha \sin \phi \sin \delta + \sin \alpha [\cos \phi_n \cdot (\tan \phi \sin \phi \sin \delta - \sin \delta \sec \phi)]$$

$$B_1 = \cos \alpha \cos \phi \cos \delta + \sin \alpha \cos \phi_n \cdot \sin \phi \cos \delta$$

$$C_1 = \sin \alpha \cos \delta \sin \phi_n$$

where α = angle of inclination of surface
 ϕ = latitude,
 δ = solar declination
 ϕ_n = azimuth of surface relative to the
meridian.

For a vertical surface,

$$S_v = S_m \cos h_o \cos (\phi_o - \phi_n) \quad \text{-(ii)}$$

where h_o = solar elevation and ϕ_o = solar azimuth.

For the particular cases of south, east/west and north-facing surfaces,

$$S_{vs} = S_m \cos h_o \cos \phi_o \quad \text{-(iii)}$$

$$S_{vw(e)} = S_m \cos h_o \sin \phi_o \quad \text{-(iv)}$$

$$S_{vn} = S_m [\sin \delta \cos \phi - \cos \delta \sin \phi \cos \Omega] \quad \text{-(v)}$$

From equation (iii), (iv) and (v),

$$S_v = S_{vs} \cos \phi_n + S_{vw(e)} \sin \phi_n \quad \text{-(vi)}$$

and for the more general case of a surface inclined at angle α it is shown that

$$S_s = S_m \cos \alpha + S_v \sin \alpha \quad \text{-(vii)}$$

Diffuse Component: The non-isotropy of sky and ground diffuse radiation is discussed and measurement-derived graphical representations of the ratio of the diffuse flux on variously inclined and orientated surfaces to the diffuse flux on a horizontal plane are presented. No algebraic formula derived from such graphs is quoted.

For daily totals of radiation, Chapter 3 quotes the formula suggested by B.A. Eisenstadt (ref 2), which assumes an isotropic sky, viz:

$$\Sigma Q_s = \Sigma S_s + \cos^2 \frac{\alpha}{2} \Sigma D_H + \sin^2 \frac{\alpha}{2} \Sigma \gamma_H$$

where Q_s is the global irradiation on an arbitrary inclined plane.

D_H is the diffuse irradiation on a horizontal surface.

γ_H is not defined in Kondratyev's publication. Presumably it is the product of the

global radiation on a horizontal surface and the ground albedo.

Data Requirements:

Diffuse irradiances on a horizontal surface, normal incidence direct irradiances, latitude of site.

Accuracy:

Not quoted, but the author does stress the inadequacy of the isotropic approximation for sky diffuse radiation in other than cloudy conditions.

Computing:

Not specified.

Further references:

1. Kondratyev, K.Ya.: Radiation in the atmosphere. Academic Press, New York (1969).
2. Eisenstadt, B.A.: On the scattered radiation income to slopes and mountain valley bottom. Trudy of Main Geophys. Observ. issue 107, pp 84 - 104 (1961).

2.8 Daily Insolation on Surfaces Tilted toward the Equator.

Reference:

ASHRAE Journal, 3, pp 53-59 (No 10, 1961)

Authors:

B.Y.H. Liu and R.C. Jordan, Departement of Mechanical Engineering, University of Minnesota.

Abstract:

A method is described which converts measurements of global irradiation on a horizontal surface into their corresponding values on a sloping plane. Although, with modification, the method is probably applicable to surfaces of any orientation, the present study is restricted to surfaces tilted toward the equator. Theoretical predictions are compared with measurements made at Blue Hill, Mass, USA.

Description:

Using a conversion factor, R, and working with long term (say monthly) means, the object of the method is to compute day-totals of irradiation on a tilted surface (orientated toward the equator) from measured day-totals of global irradiation on a horizontal plane.

Irradiation, H_t , on a tilted surface is considered as comprising 3 components - direct, diffuse and ground-reflected, each with their own conversion factors, R_D , R_d and R_p , so that

$$H_t = (H-D)R_D + D.R_d + H.R_p$$

where H and D are the global and diffuse irradiances on a horizontal plane.

$$\text{Thus, } R = \frac{H_t}{H} = (1 - \frac{D}{H}) R_D + \frac{D}{H} R_d + R_p$$

Daily ratios D/H (assuming D is not measured) are obtained from a statistical relationship between H and the extra-terrestrial irradiation, H_0 , on a horizontal surface (see ref 1). The conversion factors R_D , R_d and R_p are derived as follows:

R_D : The authors derive an approximation for R_D by geometrical consideration of the special case of a surface located just outside the earth's atmosphere, where no consideration need be given to attenuation by the atmosphere (which, being a function of hour angle, complicates the general case). This approximation is justified by showing it to be an exact fit to the situation at the earth's surface when the sun is at the equinox (ie solar declination zero).

Outside the atmosphere the solar radiation incident on a horizontal surface is entirely direct radiation and therefore

$$R_D = \frac{H_{Ot}}{H_0}$$

The instantaneous extra-terrestrial irradiances are

$$I_{Oh} = I_{On} \cos \theta_h \text{ for a horizontal plane}$$

$$\text{and } I_{Ot} = I_{On} \cos \theta_t \text{ for a tilted plane}$$

where I_{On} is the instantaneous normal incidence extra-terrestrial irradiance and θ_h , θ_t are the angles of incidence of the solar beam on horizontal and tilted surfaces. θ_h and θ_t are functions of latitude, L, solar declination, δ , and hour angle, ω :

$$\cos \theta_h = \cos L \cos \delta \cos \omega + \sin L \sin \delta$$

and noting that a surface located at latitude L, tilted β degrees from the horizontal toward the equator, is parallel to a horizontal surface located at latitude $L-\beta$,

$$\cos \theta_t = \cos (L-\beta) \cos \delta \cos \omega + \sin (L-\beta) \cdot \sin \delta$$

Daily extra-terrestrial irradiances are obtained by integrating the instantaneous irradiances over the period for which the sun is above the horizon. This is equivalent to integrating over all hour angles from $+\omega_s$ to $-\omega_s$ where ω_s is the sunset hour angle (ω_s' for a tilted surface, defining sunset as the time when $\cos \theta_t = 0$).

$$\cos \omega_s = -\tan L \tan \delta$$

$$\cos \omega_s' = -\tan (L-\beta) \tan \delta$$

Performing these integrations to obtain H_0 and H_{Ot} it is shown that

$$R_D (= \frac{H_{Ot}}{H_0}) = \frac{\cos (L-\beta)}{\cos L} \cdot \frac{(\sin \omega_s - \omega_s \cos \omega_s')}{(\sin \omega_s - \omega_s \cos \omega_s)}$$

(when $\omega_s < \omega_s'$) (i)

$$\text{or } R_D = \frac{\cos (L-\beta)}{\cos L} \cdot \frac{\sin \omega_s' - \omega_s' \cos \omega_s}{\sin \omega_s - \omega_s \cos \omega_s}$$

(when $\omega_s' < \omega_s$) (ii)

For a plane located on the surface of the earth the direct irradiance is reduced to a fraction, τ , of the extra-terrestrial irradiance by the effects of scattering and absorption; and τ is a function of hour angle. Therefore, in general, R_D can only be evaluated when this functional relationship is known.

In general:

$$R_D = \frac{\int_{\omega_s \text{ or } \omega_s'}^{\omega_s \text{ or } \omega_s'} \tau I_{On} [\cos(L-\beta) \cos \delta \cos \omega + \sin(L-\beta) \sin \delta] d(\frac{24\omega}{2\pi})}{\int_{-\omega_s}^{\omega_s} \tau I_{On} [\cos L \cos \delta \cos \omega + \sin L \sin \delta] d(\frac{24\omega}{2\pi})}$$

However, on the equinox, when $\delta = 0$ and $\omega_s = \omega'_s = \frac{\pi}{2}$, this expression reduces to

$$R_D = \frac{\cos(L-\beta)}{\cos L},$$

which is independent of τ , and is the same result as that predicted by equations (i) and (ii) setting $\delta = 0$. Thus it is shown that equations (i) and (ii), though derived for extra-terrestrial solar radiation only, do give the correct expression for R_D when the sun is at the equinox. Since the atmosphere transmission for all but cloudless days, is meagrely understood it is recommended that equations (i) and (ii) be used as a first approximation.

R_d : An isotropic sky is assumed. Then it can be shown that the ratio of diffuse irradiation on a plane inclined at angle β to the diffuse irradiation on a horizontal plane is:

$$R_D = \frac{1}{2} (1 + \cos \beta)$$

R_p : For a horizontal global irradiance I_{Th} the reflected irradiance is ρI_{Th} and this radiation is reflected diffusely. It can be shown, by integration, that the radiance reflected on to a tilted surface has an irradiance

$$I_p = \frac{(1-\cos\beta)}{2} \rho I_{Th}$$

Thus

$$R_p = \frac{I_p}{I_{Th}} = \frac{(1-\cos\beta)}{2} \rho$$

(A method for deriving an average reflectivity for ground of non-uniform reflectivity is also presented).

Data requirements:

Long term averages of daily global irradiation on a horizontal surface, (also ground reflectivity, latitude, and solar declination).

Accuracy:

No figure is quoted for the accuracy of the method, though a table compares theoretical values of R with those derived from measurements.

Computing:

Not specified.

Further references:

Liu, B.Y.H. and Jordan, R.C.: "The interrelationship and characteristic distribution of s_e and total solar radiation". Solar Energy, Vol 4, (No 3, July 1960).

2.9 The determination of Hourly Insolation on an Inclined Plane Using a Diffuse Irradiance Model Based on Hourly Measured Global Horizontal Insolation.

Reference:

Solar Energy, 19, pp 477 - 491, (No 5, 1977).

Author:

J.W. Buglar, Capricornia Institute of Advanced Education, Rockhampton, Australia 4700.

Abstract:

Using only measured hourly values of global insolation on a horizontal surface, a method has been developed for computing the corresponding hourly values of insolation at any angle and orientated in any direction. The method uses a solar radiation model in which the diffuse component is calculated from the global horizontal radiation using three different relationships: the appropriate equation is selected according to the value of the ratio of measured hourly global insolation to hourly global insolation computed for clear sky conditions. The method has been checked using measured hourly values in Melbourne over a 5-year period of insolation both on a horizontal surface and on a plane inclined at 38° to the horizontal facing North. The differences between the computed hourly values and the measured hourly values are found to be approximately normally distributed about zero with a standard deviation of 0.16 MJm^{-2} . This method is particularly useful for predicting the heat output of inclined solar flat plate collectors when only measured global horizontal insolation is available, which is often the case. Good agreement was found between the predicted output of a typical collector using measured 38° insolation and the computed hourly values using this method. Since the method has been checked only against Melbourne data it should be applied elsewhere with caution, but it is believed to have general application.

Description:

The aim is to convert measured hourly values of global irradiation on a horizontal surface into the corresponding hourly values for an inclined surface of orientation, for any sky condition. The separate direct and diffuse components for a horizontal surface obtained as set out below are multiplied by appropriate geometrical factors to obtain inclined surface values.

The direct normal incidence irradiance from a cloudless sky (I_N) is obtained from curves (as functions of atmospheric precipitable water and dust content) given by Rao and Seshadri (ref 1).

The diffuse sky radiation is subdivided into 2 components namely:

- (i) a circumsolar component, taken as being 5% of the direct normal irradiance (C), and
- (ii) a uniform background irradiance, (D).

For a cloudless sky, the latter component is obtained from the relationship:

$$D = 16.0 \alpha^{0.5} - 0.4 \alpha \quad \text{Wm}^{-2}$$

where α is the solar altitude in degrees.

The hourly global irradiation (T) for clear sky conditions can then be computed by simple geometry and summing the direct, circumsolar and background components.

Knowing T and also the measured hourly value, G, of global irradiation on a horizontal surface, the background diffuse irradiation for skies with cloud is computed from one of the following three equations.

- 1) $D = 0.94G$ for $0 < G/T < 0.4$
- 2) $D/G = \frac{(1.29 - 1.19 G/T)}{(1.00 - 0.334 G/T)}$ for $0.4 < G/T < 1.0$
- 3) $D/G = 0.15G$ for $G/T > 1.0$

For an inclined surface we have a fourth component - that of ground-reflected radiation, J. Thus for a surface inclined at angle θ and exposed for a time t_0 , the global irradiation is

$$G_\theta = \int_0^{t_0} (I_\theta + D_\theta + J_\theta + C_\theta) dt$$

The direct and circumsolar contributions are considered together ($I + C = KI$). Then, using standard relationships:

$$G_\theta = \int_0^{t_0} [k I_N \cos i + D_H (1 + \cos\theta)/2 + G_H \rho (1 - \cos\theta)/2] dt$$

where ρ = albedo.

Using hourly average values, and allowing for the fact that the sun may only be shining for a time t within the period,

$$G_\theta = k I_N t \cos i + D_H (1 + \cos\theta)/2 + G_H \rho (1 - \cos\theta)/2$$

For horizontal surface in particular

$$G_H = k I_N t \sin \alpha + D_H$$

from which a value of t can be determined.

Data requirements:

Hourly global irradiation on a horizontal surface, precipitable water (in cm) and atmospheric dust content (in particles/cm³).

Accuracy:

A comparison with measured values on a 38° inclined surface facing North, for each daylight hour in a 5-year period at Melbourne gave errors approximately normally distributed about zero with a standard deviation of 0.16 MJm⁻² (ground reflected radiation was not included).

Computing:

A computer program for the computations was written (but not given in the paper). The direct normal incidence curves of Rao and Seshadri were used in the matrix form suitable for computer usage, given by Spencer (ref 2).

Further references:

- 1) K.R. Rao, T.N. Seshadri: Solar Insolation Curves. Indian J Met Geophys, 12, pp 267 - 272 (No 2, 1961).
- 2) J.W. Spencer: Computer estimation of direct solar radiation on clear days. Solar Energy, 13, pp 437 - 438, (No 4, 1972).

2.10 Direct and Scattered Radiation Reaching the Earth as Influenced by Atmospheric Geographical and Astronomical Factors.

Reference:

"Solar Radiation", Elsevier Publishing Company, 1966, Chapter 4.

Authors:

1) N. Robinson, Solar Physics Laboratory, Institute of Technology, Haifa, Israel.

2) W. Schüepp, Meteorological Observatory, Basel, Switzerland.

Abstract:

Chapter 4 of this book discuss the estimation on a theoretical basis of direct, diffuse and global irradiances on horizontal and inclined surface for both clear sky conditions and skies with cloud. Data obtained at various observatories are reviewed and where possible compared with theoretical expectations. The basic steps of the method are set out below but involve the use of tables and graphs to be found in the book. It is pointed out that at the time of writing, some of the physical constants involved are not known to a sufficient degree of accuracy.

Description:

1) For cloudless skies:

The direct component at normal incidence to the sun is given by Ångström's turbidity formula:

$$I' = (R_0/R)^2 \int_0^{\infty} I'_{0\lambda} 10^{-(m \alpha_A + m_r \alpha_d + \alpha_w) \lambda} d\lambda$$

where $(R_0/R)^2$ is the mean earth-sun distance correction factor.

$I'_{0\lambda}$ is the monochromatic normal incidence irradiance outside the earth's atmosphere.

m is the air mass number corrected for pressure by $m = \frac{p m_r}{1000}$ where p = pressure in mb, m_r is the relative air mass along the direction of the solar elevation h ,

α_A is the Rayleigh extinction coefficient (corrected for refraction and non-isotropy in air molecules),

α_d is the extinction coefficient for dust,

α_w is the absorption coefficient for water vapour, ozone, carbon dioxide and other gases.

Note that $\alpha_A = 0.00386 \lambda^{-4.05}$

$$\alpha_d = \beta (2 \lambda)^{-\alpha}$$

where β is the Ångström-Shüepp turbidity coefficient and α is approximated by a fixed value of 1.5.

Tables given enable the computations of I' when solar elevation, precipitable water content of the atmosphere and turbidity coefficient are known. For an inclined surface the direct component is known from geometry:

$$I'_E = I' \cos i$$

where i is the angle between the normal to the surface and the solar beam.

The diffuse component on a horizontal surface is given by Albrecht's formula (ref 1):

$$D' = k(I'_w - I') \sin h$$

where h is the solar elevation angle, $k = 0.5 \sin^{\frac{1}{2}} h$ and I'_w is the direct component allowing for absorption but not for scattering effects. Then assuming an isotropic sky the diffuse sky component on a surface of inclination E to the horizontal is given by

$$D'_{ES} = D' \cos^2 E/2$$

The ground reflected radiation on an inclined surface is allowed for by adding

$$D'_R = (I'_H + D') R (1 - \cos^2 E/2)$$

where I'_H is the direct irradiance on a horizontal surface and R is the ground reflection coefficient (albedo).

The global irradiation on a surface of arbitrary orientation is then given by

$$T'_E = I'_E + D'_{ES} + D'_R$$

2) Cloudy skies

The degree of cloudiness is estimated from the relative sunshine duration n/N and the diffuse sky component is subdivided into two parts: - one from the clear portion of the sky, and the other from the cloudy part. The equations for I_E' , D_{ES}' and D_R' then become

$$I_E' = I' n/N \cos i$$

$$D_{ES}' = n/N R (I_W' - I') \sin h \cos^2 E/2 +$$

(from clear portion)

$$+ f(n/N) [I_H' + k(I_W' - I') \sin h] \cos^2 E/2$$

(from cloudy portion)

$$D_R' = \frac{n}{N} \cdot [I_H' + k(I_W' - I') \sin h] +$$

$$+ f\left(\frac{n}{N}\right) [I_H' + k(I_W' - I') \sin h] \cdot$$

$$\cdot R (1 - \cos^2 E/2)$$

The terms I' , I_W' , I_H' on the rhs of these formulae refer to the cloudless state as in § 1.

$f(n/N)$ is a constant of proportionally dependent upon n/N . Tables are given which suggests values of $f(n/N)$.

Data Requirements:

Solar elevation and azimuth, daily or hourly sunshine durations, altitude and atmospheric pressure at the site. Also required are the precipitable water content of the atmosphere, Angström's turbidity coefficient, ground albedo, but guidance about the values to choose or how to calculate them are given in the text of the chapter.

Accuracy:

No figure quoted.

Computing:

Not specified.

Further references:

1) Albrecht, F.: Arch Meteorol Geophys. Bioklimatol Ser B, 3, 220 (1951).

2.11 The Solar Energy Incident on a Plane at the Earth's Surface. Situation in Belgium

Reference:

Arch Met Geoph Biokl, Ser B, Vol 25, pp 135 -150 (No 2, 1977).

Authors:

A. De Vos and G. De Mey, Laboratorium voor Elektronika en Meettechniek, Gent, Belgium.

Abstract:

The Solar Energy incident yearly on a plane surface at the Earth's surface, is calculated as a function of the fixed orientation of the plane. Distinction was made between direct sun radiation diffuse sky radiation and diffuse ground-reflected radiation. Statistical data for the clouding was taken into account. The mathematical models have been kept as simple and as general as possible. They have been extensively tested for the case of Belgium. Calculations for the Belgian situation give an optimum orientation for solar power converters.

Description:

This paper deals with annual irradiation. Let E be the annual radiation falling on the plane (in $J m^{-2}$). E is normalised with respect to $E_0 = P_0 T_0$, the product of solar irradiance (P_0 in $W m^{-2}$) for zero air mass, and the period of 1 year (T_0 in seconds).

E is less than E_0 for 3 main reasons:

(i) The plane is illuminated for a period, T_1 which is less than T_0

(ii) The plane is illuminated by a solar irradiance P which is less than P_0 because of absorption by atmospheric gases, clouds and haze.

(iii) For most of the time the plane is not orientated normally to the sun's rays, introducing a correction factor, c , given by $c = \cos i$ for $\cos i > 0$ and $c = 0$ for $\cos i < 0$ (surface in shadow) where i is the incidence angle of the radiation on the plane.

Direct component

Let $m = \text{air mass} = \sec Z_0$, where $Z_0 = \text{solar zenith angle}$.

$k = \text{extinction factor of the atmosphere}$

W = a statistical weighting factor to account for attenuation by clouds and haze.

Then the direct solar irradiance is approximated by

$$P_1 = P_0 W \exp(-km) \quad (i)$$

Diffuse sky component

Following from equation (i), the direct irradiance on a horizontal plane at the earth's surface is $WP_0 \cos Z_0 \exp(-km)$. Therefore the remaining fraction of P_0 , say the authors, is absorbed by the atmosphere and clouds. This fraction is given by $P_0 \cos Z [1-W \exp(-km)]$ and this power is considered to be partly converted into non-radiant forms of energy and partly remitted in all directions. Thus, a fraction of this energy reaches the earth as diffuse radiation, and a sky parameter, s_1 is introduced such that the diffuse irradiance on a horizontal plane is $S P_0 \cos Z [1-W \exp(-km)]$ ($0 < S < 1$)

For a plane inclined at angle Z to the horizontal, only a fraction of this power is received, this fraction being

$$\iint \cos \theta \, d\Omega = \frac{1}{2} (1 + \cos Z)$$

$$\iint_{\Omega_1} \cos \theta \, d\Omega$$

upper hemisphere

where Ω_1 is the part of the sky seen by the inclined plane. This statement assumes that the sky is isotropic. The diffuse sky irradiance for a plane inclined at angle Z is therefore:

$$P_2 = S P_0 \cos Z_0 [1-W \exp(-km)] \frac{1}{2} \cdot (1 + \cos Z) \quad (ii)$$

Ground reflected component

For ground albedo the power of ground reflected radiation, assumed isotropic, is

$$\gamma [WP_0 \cos Z \exp(-km)] +$$

(horizontal component of direct irradiance)

$$+ SP_0 \cos Z [1-W \exp(-km)] =$$

(diffuse sky irradiance on a horizontal plane)

$$= \gamma P_0 \cos Z_0 [S + (1-S) W \exp(-km)]$$

A plane inclined at angle Z will receive a fraction of this power density:

$$\iint \cos \theta \, d\Omega = \frac{1}{2} (1 - \cos Z)$$

$$\iint_{\Omega_2} \cos \theta \, d\Omega$$

lower hemisphere

where Ω_2 = part of earth's surface seen by the inclined plane.

The ground reflected irradiance for an inclined plane is therefore:

$$P_3 = \gamma P_0 \cos \theta_0 [S + (1-S) W \exp(-km)] \frac{1}{2} \cdot (1 - \cos Z) \quad (iii)$$

The annual direct, sky diffuse and ground reflected irradiances on an inclined plane can now be written as:

$$E_1 = \sum_{n=1}^{365} J(n) \int_{t_1(n)}^{t_2(n)} P_1(A, Z, n, t) \, dt$$

$$E_2 = \sum_{n=1}^{365} J(n) \int_{t_1(n)}^{t_2(n)} P_2(A, Z, n, t) \, dt$$

$$E_3 = \sum_{n=1}^{365} J(n) \int_{t_1(n)}^{t_2(n)} P_3(A, Z, n, t) \, dt$$

where: A = azimuth angle of the plane

n = the day of the year

$t_1(n), t_2(n)$ = time of sunrise and sunset on nth day

J(n) = correction factor to allow for the annual oscillation in the earth-sun radius.

Because the irradiance, P, does not exactly follow the formula

$$P(Z_0) = P_0 \exp(-k \sec Z_0)$$

a value for P_0 more appropriate than the solar constant was chosen in order to match experimental curves as closely as possible with this expression. The authors obtained P_0 and k from curves proposed by Dogniaux (ref 1) which relate P to air mass. In the range of solar zenith angles of interest these curves approximate to straight lines. Hence P_0 was determined by extrapolating to air mass = zero, and k was taken as the slope of the linear portion. A compromise between Dogniaux curves for urban and industrial regions was used in the paper.

The weighting factor, W , was taken as the fraction of the possible solar irradiance actually measured, on average, on a particular date. W was assumed independent of time of day. The experimental histogram showing W as a function of day of year was fitted by a sine curve.

$$W(n) = W_1 + W_2 \cos \frac{2\pi n}{365}$$

and the values of W_1 and W_2 evaluated.

The method of obtaining the sky parameter, S , is not elaborated in the paper, but the reader is referred to ref 2, from which a value of 0.4 was chosen.

Data Requirements:

Latitude of site, statistical data for the average daily irradiation on a horizontal surface at different times of year (the authors divided the year into 36 parts) in order to evaluate the clouding function, $w(n)$.

Accuracy:

Measurements on inclined surfaces were not available for comparison with computed values. Computed direct, diffuse and total daily irradiations (annual average) on a horizontal plane gave good agreement with the mean (1971-75) measured values at Ukkel. The direct component in summer was slightly over-estimated.

Computing:

A computer was used to calculate, and plot on polar diagrams, values of the normalised irradiances ($e = E/E_0$) for different angles of azimuth and inclination of surface. Details of the program are not given.

Further references:

1. Dogniaux, R.: Variations géographiques et climatiques des expositions énergétiques solaires sur des surfaces réceptrices horizontales et verticales. Inst roy Météorologiques de Belgique, Brussels, 1975.

2. Dave, J.P. et al: Computation of Incident Solar Energy, IBM Research and Development, 19, 539 - 549 (1975).

2.12 Solar Radiation at a Site of Known Orientation on the Earth's Surface

Reference:

J Appl Met, Vol 15, pp 651 - 656, (No 6, 1976).

Author:

Revfeim, K.J.A. of Biometrics Section, Ministry of Agriculture and Fisheries, Wellington, New Zealand.

Abstract:

Attenuation of the direct solar beam as a function of hour angle is expressed as a Fourier cosine series. A simple procedure is derived for calculating the direct component of daily radiation at a site with known latitude and orientation. This paper is concerned only with the direct component of radiation.

Description:

The normally incident direct radiation is calculated from the solar constant by applying to it a single transmission coefficient, t , (allowing for both scattering and absorption effects - see de Lisle, 1966, ref 1).

This transmission coefficient is raised to the power of the optical air mass $m(Z)$ at a particular zenith angle, Z , in order to calculate the fraction of the solar beam which is transmitted. The author explains why a closer approximation to $m(Z)$ is required than the frequently used $m(Z) = \sec Z$ in Lambert's formula (ref 2), viz:

$$m_L(Z) = cr \left[\left(\cos^2 Z + \frac{2u}{r} + \frac{u^2}{r^2} \right)^{\frac{1}{2}} - \cos Z \right]$$

where r = earth radius

u = the height of the atmosphere when
its density is assumed uniform
(taken as 7 km - derivation given)

c = the density decay rate of the at-
mosphere.

It is shown that

$$c = \frac{1}{u} \left[1 + \frac{u}{r} + \text{higher powers of } \frac{(u)}{(r)} \right]$$

and that

$$t^m[Z(h)] = \sum_{j=0}^{j=\infty} c_j \cos jh$$

where h is the hour angle.

and also that c_j tends to zero as j tends to infinity.

Having evaluated the normal incidence component, Q' , the direct irradiance Q'_n on a surface of arbitrary orientation is calculated using trigonometrical relationships. It shows that

$$Q'_n = Q' [\sin \phi^* \sin \delta + \cos \phi^* \cos \delta \cdot$$

$$\cdot \cos (h-g)]$$

where ϕ^* is an effective latitude for the particular orientation of the surface, defined by

$$\sin \phi^* = \sin \phi \cos i - \cos \phi \sin i \cos b$$

where ϕ = the latitude
 i = the inclination to the horizontal
 b = azimuth of plane with respect to South.
 δ = solar declination.

and g, termed the "daylight time shift" is defined by

$$\sin g = \frac{\sin i \sin b}{\cos \phi^*}$$

Data requirements:

(Direct component only). Latitude, precipitable water content of the atmosphere (for evaluation of the transmission coefficient, t, as given by Houghton, ref 3).

Accuracy:

Not tested.

Computing:

Not specified.

Further references:

1. de Lisle, J.F.: Mean daily insolation in New Zealand. NZ J Sci, 9, pp 992 - 1005, (1966).
2. Sikhov, S.L.: Computation of solar radiation characteristics. Gidrometeor Izd, Leningrad, (1968). See p 24.
3. Houghton, H.G.: The annual heat balance of the Northern Hemisphere, J Meteor, 11, pp 1 - 9, (1953).

2.13 Hourly Solar Radiation Data for Vertical and Horizontal Surfaces on Average Days in the United States and Canada.

Reference:

US National Bureau of Standards, Building Science Series No 96 (April 1977).

Authors:

Kusuda, T. and Ishii, K.: Center for Building Technology, Institute for Applied Technology, National Bureau of Standards, Washington DC 20234, USA.

Abstract:

Hourly radiation data for walls and roofs under "average" solar conditions are computed in order to enable estimation of the effect of solar radiation on a building and its heating/cooling air conditioning system over a heating/cooling season. A description of the method is given but the larger part of the paper is devoted to extensive tables of such data for 80 sites in the US and Canada. (Data is also tabulated for a new parameter called the sol-air temperature for glass).

The aim is to compute average hourly values of global irradiation on vertical surfaces for each month of the year at a given site, using the monthly average of daily global irradiation on a horizontal surface at that site.

Description:

1) The method is basically that given by Liu and Jordan (refs 1, 2 and 3) but with a small modification in that the equation

$$\bar{I}_{DV} = \frac{\bar{I}_{Dh}}{\cos \theta_H} \cos \theta_V$$

is replaced by

$$\bar{I}_{DV} = \bar{I}_{DN} \cos \theta_V$$

where

$$\bar{I}_{DN} = \bar{A} e^{-[B/\cos \theta_H]}$$

(A and B are defined in § 2 step (vii) and (viii)), \bar{I}_{DV} and \bar{I}_{Dh} are average hourly direct irradiances on vertical and horizontal surfaces making angles of θ_V and θ_H with the normal incidence direction (see steps (viii) to xii in § 2). This is to prevent the model giving unduly large values for \bar{I}_{DV} close to sunrise and sunset ($\cos \theta_H \rightarrow 0$).

2) The steps of the procedure are therefore as follows:

(i) Daily total extra-terrestrial irradiation on a horizontal surface is calculated from

$$H_0 = \frac{24}{\pi} r I_{SC} (\cos L \cos S \sin W_s +$$

$$+ W_s \sin L \sin S)$$

where r is the ratio of solar irradiance at normal incidence outside the earth's atmosphere to the solar constant I_{SC} (monthly average values are supplied), L = latitude in radians, S = solar declination angle in radians (monthly average values are supplied), W_s = sunrise hour angle in radians, obtained by

$$\cos W_s = -\tan L \tan S$$

(ii) The ratio \bar{K}_T , of monthly average daily global irradiation on a horizontal surface \bar{H} to the value of H_0 computed above is evaluated.

(iii) Knowing \bar{K}_T the value of \bar{K}_d , the ratio of monthly average diffuse irradiation on a horizontal surface to H_0 , is obtained from a table derived from refs 3 and 4.

(iv) The monthly average daily diffuse irradiation is then

$$\bar{D} = K_d \cdot H_0$$

(v) The value of factor r_d is then calculated from

$$r_d = \frac{\pi (\cos W - \cos W_s)}{24 (\sin W_s - W_s \cos W_s)}$$

where W = hour angle corresponding to a given hour of day (radians).

(vi) The average hourly diffuse irradiation on a horizontal surface is then

$$\bar{I}_{dh} = r_d \cdot \bar{D}$$

(vii) The value of a factor f, is taken from a table for a given month and latitude.

$$[f = \frac{r \cdot I_{SC}}{G} (\cos L \cos S \sin W_s + W_s \sin L \sin S)]$$

and

$$G = \int_0^{W_s} e^{-[B/\cos \theta_h]} \cos \theta_h dW$$

where B is the atmospheric extinction coefficient.]

(viii) Using $\bar{A} = (\bar{K}_T - K_D) \cdot f$, the value of \bar{A} is calculated. \bar{A} is the multiplying coefficient in the equation

$$\bar{I}_{DN} = \bar{A} e^{-[B/\cos \theta_h]}$$

which is derived from the method of ASHRAE (ref 5).

(ix) The hourly values of \bar{I}_{DN} , the direct component on a vertical surface orientated normally to the solar beam ($\cos \theta_h = 1$) is computed from the equation in step (viii)

(x) Average hourly direct irradiation on a horizontal surface is then

$$\bar{I}_D = \bar{I}_{DN} \cos \theta_h$$

(xi) Average hourly global irradiation on a horizontal surface is

$$\bar{I}_{Th} = \bar{I}_{Dh} + \bar{I}_{dh}$$

(xii) Average hourly direct irradiation on a vertical surface is calculated as

$$\bar{I}_{DV} = \bar{I}_{DN} \cos \theta_v$$

(xiii) Average hourly diffuse irradiation on a vertical surface is approximated by

$$\bar{I}_{dv} = \bar{I}_{dh}/2$$

(xiv) Average hourly ground reflected irradiation on a vertical surface is approximated by

$$\bar{I}_{gv} = \rho g \cdot \bar{I}_{Th}$$

where ρg is the ground reflectance.

(xv) Average hourly global irradiation on the vertical surface is then given by

$$\bar{I}_{Tv} = \bar{I}_{Dv} + \bar{I}_{dv} + \bar{I}_{gv}$$

Data requirements:

Monthly average daily totals of global irradiation on a horizontal surface, average monthly atmospheric extinction coefficients, ground reflectance.

Accuracy:

Not quoted, though sample results are compared, graphically, with results from Liu and Jordan's method (refs 1, 2 and 3).

Computing:

A computer programme has been written for this procedure: the programme itself (in FORTRAN) is listed as an appendix.

Further references:

- 1) Liu, D.Y.H. and Jordan, R.C.: The inter-relationships and characteristic distribution of direct, diffuse and total solar irradiation. *Solar Energy* 4, pp 1 - 19 (No 3, 1960).
- 2) Liu, D.Y.H. and Jordan, R.C.: Daily insolation on surfaces tilted toward the equator. *ASHRAE Journal* 3, pp 53 - 59 (No 10, October 1961).
- 3) Liu, D.Y.H. and Jordan, R.C.: A national procedure for predicting the long-term average performance of flat-plate solar energy collectors. *Solar Energy* 7, pp 53 - 74 (No 2, 1963).

4) Liu, D.Y.H. and Jordan, R.C.: Availability of solar energy for flat-plate solar heat collectors. *Low Temperature Engineering Applications of Solar Energy* Chapter 1, pp 1 - 18 (1967).

5) Air-Conditioning Cooling Load, *ASHRAE Handbook of Fundamentals*, Chapter 22 (N York, 1967).

2.14 Estimating Solar Radiation on Mountain Slopes.

Reference:

Agricultural Meteorology 12, pp 329 - 336 (1973).

Authors:

1. Lloyd W Swift. Coweeta Hydrologic Lab. Southeastern Forest Experimental Station. USDA Forest Service, Franklin NC USA.

2. Kenneth R Knoerr, School of Forestry, Duke University, Durham NC USA.

Abstract:

A method is suggested by which daily totals of global irradiation on a mountain slope may be estimated from daily total of global irradiation on a horizontal surface measured at a site close enough to have similar sky conditions. Two modifications to the method are suggested.

Description:

The global irradiation on a sloping surface is estimated by multiplying the global irradiation on a nearby horizontal surface by the ratio of theoretical irradiation on the sloping and horizontal surfaces for a transparent atmosphere (simply a geometrical ratio) ie

$$T_E = T \left(\frac{T_{as}}{T_a} \right)$$

Tables of T_a and T by Frank and Lee (ref 1) were used by the authors to compute this ratio, the "slope factor", which varies over the year according to the value of the solar declination.

Modification (i): The measured global irradiation on a horizontal surface was split into direct and diffuse components by assuming that

$$\frac{I_H}{T} = \frac{T}{T_a}$$

where I_H is component of direct radiation on a horizontal surface. The slope factor was then applied only to the direct component and the diffuse component (assumed the same for all slopes) added to the result

$$T_E = I_H \left(\frac{T_{as}}{T} \right) + D$$

where D is the diffuse radiation on a horizontal surface? (this is not clear from the published paper).

Modification (ii): Topographic shading factors are added to the result in order to correct for the excess shading of the horizontal measuring instrument over that of the mountain slope. These can either be evaluated by experiment (minimising the mean differences between measured and estimated values on slope) or by methods proposed by Baumgartner, Lee and Ohmura (refs 2 - 4).

Data requirements:

Daily totals of T at a nearby site (or preferably half-daily totals to account for changes in cloud amount before and after noon).

Accuracy:

The majority of half-daily totals in a 26-day test for N and S-facing surfaces inclined at 37° lay within ± 10% of measured value, using modification (i).

Computing:

Not specified.

Further references:

1. Frank, E.C. and Lee, R.: Potential Solar Beam irradiation on slopes, tables for 30° to 50° latitude. US Dept of Agr Forest Serv Rocky Mountain Range Expt Sta, Res. Pap RM-18 (1966).
2. Baumgartner, A.: Gelände und Sonnenstrahlung als Standortfaktor am Gr. Falkenstein

(Bayerischer Wald) Forstweiss Centralbe. 79 pp 286 - 297 (1960).

3. Lee, R. and Baumgartner, A.: The topography and insolation climate of a non-mountainous forest area. Forest Science 12, pp 258 - 267 (1966).

4. Ohmura, A.: The influence of the sky-line on the incidence of direct short-wave radiation. McGill Univ Dept Geog Climatol Bull. 7, pp 17 - 24 (1970).

2.15 Define, Develop and Establish a Merged Solar and Meteorological Computer Data Base.

Reference:

Canadian Climate Centre Report No. 80-8 Atmospheric Environment Service, Downsview, Ontario, Canada.

Authors:

James F. MacLaren Limited
Hooper and Angus Associates Limited
Dr. John Hay (University of British Columbia)
Dr. John Davies (McMaster University)
Dr. D.C. McKay (Canadian Climate Centre)
Scientific Authority

Enquiries can be addressed to Dr. D.C. McKay

Canadian Climate Centre
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4905 Dufferin St.
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Abstract:

The overall objective of the study consisting of three parts was to define, develop and establish a merged solar and meteorological data base for Canada.

The sub-objective of Part 1 was to expand the existing solar data base of the Atmospheric Environment Service by estimating hourly direct, diffuse and global radiation on horizontal surfaces for a ten year period for a number of hourly weather reporting stations by using an appropriate simulation model or models. Six models were selected for evaluation using statistical comparisons of model estimates and measured test data from a

number of stations. A model developed by Dr. John Davies of McMaster University, MAC 3 model, was selected and used to create an hourly data base of solar radiation on a horizontal surface for a number of locations in Canada.

The sub-objective of Part 2 of the study was to develop a data base of solar radiation availability on inclined surfaces of varying angles and azimuths. Three models which estimate solar radiation on sloping surfaces given solar radiation on horizontal surfaces were evaluated by comparing statistical deviations of model estimates from measured test data. As a result of this comparison, a model developed by Dr. John Hay at the University of British Columbia, the anisotropic model, was selected for generating slope radiation data for Canadian locations.

The sub-objective of Part 3 of the study was to define and develop a merged solar and meteorological data base. As well as providing the hourly means and standard deviations by month for each of global, sky diffuse and direct solar radiation on a horizontal surface and daily means and standard deviations by month for orientations of S, SE, SW, W, E, and N and tilts of 30°, 60° and 90° for each of global, sky diffuse, and direct solar radiation were produced for each location.

Descriptions:

The models described are:

(i) The MAC 3 model used to compute global, direct and diffuse solar radiation on a horizontal surface.

(ii) The anisotropic model used to compute global, direct, sky diffuse and reflected solar radiation on sloping surfaces.

(i) MAC 3

Direct beam, diffuse and global irradiances under cloudless skies are calculated by:

(a) Direct beam

The direct beam irradiance is given by:

$$I_0 = I(0) \cos \theta [T_o(U_o m_r) T_R(m_r) - a_w(U_w m_r)] \cdot$$

$$T_a(m_r)$$

in which θ is the solar zenith angle and T_o , T_R and T_a are transmissivities after absorption by ozone, Rayleigh scattering and extinction by aerosol and a_w is the absorptivity of water vapour. Vertical path lengths through ozone (U_o) and water (U_w) become the beam path lengths when multiplied by the relative optical air mass.

(b) Diffuse

Diffuse irradiance is expressed as the sum of components from Rayleigh (D_R) and aerosol (D_A) scatter. Assuming that scattering is isotropic in both forward and backward hemispheres and that water vapour attenuates only the direct beam the components are expressed as:

$$D_R = I(0) \cos \theta T_o(U_o m_r) [1 - T_R(m_r)] T_a(m_r) / 2$$

$$D_A = I(0) \cos \theta [T_o(U_o m_r) T_R(m_r) - a_w(U_w m_r)] \cdot$$

$$[1 - T_a(m_r)] w_o B_a$$

In equation for D_A , it was assumed that the scattered radiation is not only isotropic over the upward and downward hemispheres but that it was equal in both. Hence, half of the total scattered radiation is directed downward. The transmissivity after extinction by aerosol T_a is evaluated for $m_r' = 1.66$ since the incident irradiance is diffused prior to attenuation by aerosol. The single scattering albedo w_o allocates to scattering the fraction of the radiation that is attenuated by aerosol and B_a is the fraction of the total scattered radiation that is directed downward. The sum of D_R and D_A represents the primary diffuse component.

Cloudy Sky Irradiances

Hourly meteorological observations in Canada provide information on cloud amount and type for up to four layers in the atmosphere. Cloud amount in each layer is assumed to be distributed uniformly over the sky. The transmissivity for the i th layer T_{ci} con-

taining cloud amount C_i with a transmissivity t_i is

$$T_{C_i} = 1 - C_i + t_i C_i$$

where $1 - C_i$ is the transmissivity of the cloudless portion of the sky through which direct beam and diffuse radiation passes without attenuation by the layer and $t_i C_i$ is the transmissivity of the clouded portion. Total cloud transmissivities for all layers is the product of the layer transmissivities:

$$T_c = \prod_{i=1}^n T_{C_i}$$

Secondary diffuse effects or multiple scattering between the ground and cloud base are incorporated as a function of surface albedo α_s and the albedo of the atmosphere for surface reflected radiation α_b . The α_b term is the sum of components due to Rayleigh scatter $\alpha_b(R)$, scatter by aerosol $\alpha_b(A)$, and reflection from cloud bases $\alpha_b(C)$. To arrive at an expression for α_b it is assumed that $\alpha_b(R)$ applies only to the cloudless portion of the sky; that $\alpha_b(A)$ applies to the atmosphere below cloud base and that $\alpha_b(C)$ can be expressed as the product of an average cloud albedo $\bar{\alpha}_c$ and total cloud amount C_T . Then

$$\alpha_b = \alpha_b(R) (1 - C_T) + \alpha_b(A) + \bar{\alpha}_c C_T$$

where $\alpha_b = 0.0685$ and

$$\alpha_b(A) = [1 - T_a(m'_R)]_{w_o} [1 - B_a(m'_R)]$$

General equations for irradiances under all cloud conditions can be written as

$$I = I_0 (1 - CO)$$

$$K\downarrow = (I_0 + D_R + D_A) \prod_{i=1}^4 [(1 - C_i) + t_i C_i] / (1 - \alpha_s \alpha_b)$$

and

$$D = K\downarrow - I$$

where CO , is total cloud opacity, I is the direct component, $K\downarrow$ is the global irradiance and D is the diffuse component.

Parameterizations

(1) Absorption by ozone

A constant value of 3.5 mm for ozone amount has been assumed in this study. Data from McClatchey et al (1971) suggest that this value is close to the average for mid-latitude and subarctic regimes in all seasons. The value used is not of critical importance since attenuation of the solar beam by ozone is small (3%). Following Lacis and Hansen (1974), the transmissivity after absorption by ozone is expressed by

$$T_o(X_1) = 1 - a_o(X_1)$$

$$a_o(X_1) = \frac{0.1082X_1}{(1+13.86X_1)^{0.805}} + \frac{0.00658X_1}{1+(10.36X_1)^3} + \frac{0.002118X_1}{1+0.0042X_1+0.00000323X_1^2}$$

where $X_1 = U_o m_R$

(2) Absorption by water vapour

Again, following Lacis and Hansen (1974), the absorptivity by water vapour is given by

$$a_w(X_2) = 0.29X_2 / [(1+14.15X_2)^{0.635} + 0.5925X_2]$$

where $X_2 = U_w m_R$ and U_w is calculated from Paltridge and Platt (1976)

$$U_w = U'_w \left(\frac{P}{P_o}\right)^{0.75} \left(\frac{T_o}{T}\right)^{0.5}$$

where T is the surface temperature ($^{\circ}K$) and $T_o = 273^{\circ}K$ and P is the station pressure and P_o is the standard pressure (101.3 Kpa).

$$U'_w = \exp(2.2572 + 0.05454 T_d)$$

where T_d is the station dew point temperature in $^{\circ}C$. This provides values of precipitable water in millimeters.

(3) Transmissivity after Rayleigh scatter

A procedure by Elterman (1968) to calculate spectral values of the Rayleigh scatter optical depth has been revised by Dr. J.A. Davies using an updated depolarization factor as recommended by Hoyt (1977) and the

extraterrestrial solar spectrum data of Thekaekara and Drummond (1971). Spectrally integrated values of $T_R(m_r)$ were calculated and can be interpolated readily from the table given below.

m_r	0.5	1	1.2	1.4	1.6	1.8
$T_R(m_r)$	0.9385	0.8973	0.8830	0.8696	0.8572	0.8455
m_r	2.0	2.5	3.0	3.5	4.0	4.5
$T_R(m_r)$	0.8344	0.8094	0.7872	0.7673	0.7493	0.7328
m_r	5.0	5.5	6.0	10.0	30.0	
$T_R(m_r)$	0.7177	0.7037	0.6907	0.6108	0.4364	

(4) Transmissivity after extinction by aerosol

For the MAC 3 model

$$T_a(m_r) = 1.0$$

(5) Aerosol parameters W_o and B_a

The single scattering albedo approaches unity for an aerosol which mainly scatters. In urban areas, where aerosols may be significant absorbers, values may be smaller. In this model $W_o = 0.98$. The ratio of forward to total scatter $B_a(m_r)$ is interpolated as a function of air mass from data given by Robinson (1962). Robinson's values are listed in the table below.

θ	0	25.8	36.9	45.6	53.1
m_r	1.00	1.11	1.25	1.43	1.66
$B_a(m_r)$	0.92	0.91	0.89	0.86	0.83
θ	60.0	66.4	72.5	78.5	90.0
m_r	2.00	2.50	3.33	5.02	∞
$B_a(m_r)$	0.78	0.71	0.67	0.60	0.60

(6) Cloud transmissivities

As with all of the cloud layer models the empirical data of Haurwitz (1948) for Blue Hill are used to define cloud type transmissivities t_i using

$$t_i = A_i \exp(-B_i m_r)$$

Values of A_i and B_i for the cloud types recorded at Canadian meteorological stations are listed in the table below.

AES cloud type	Sym- code	bol	Type	A _i	B _i
0			None		
1	AC		Alto cumulus	0.556	0.053
2	ACC		Alto cumulus Castellanus	0.556	0.053
3	AS		Alto stratus	0.413	0.004
4	CC		Cirrocumulus	0.923	0.089
5	CS		Cirrostratus	0.923	0.089
6	CI		Cirrus	0.871	0.020
7	CB		Cumulonimbus	0.119	-0.226
8	CU		Cumulus	0.368	0.045
9	CF		Cumulus Fractus	0.368	0.045
10	SF		Stratus Fractus	0.252	0.100
11	TCU		Towering Cumulus	0.368	0.045
12	NS		Nimbostratus	0.119	-0.226
13	SC		Stratocumulus	0.368	0.045
14	ST		Stratus	0.252	0.100
15	F		Fog	0.163	-0.031
16			Obstruction other than fog	0.163	-0.031

(7) Surface albedo

A temperature-based algorithm has been used to calculate surface albedo in the model. The equations for surface albedo α_s , is given as

$$\alpha_s = \alpha_s(L) \text{ when } T \leq T(L)$$

$$\alpha_s = \frac{[T - T(L)] [\alpha_s(H) - \alpha_s(L)]}{T(H) - T(L)} + \alpha_s(L)$$

when $T(L) < T < T(H)$

$$\alpha_s = \alpha_s(H) \text{ when } T \geq T(H)$$

where α_s = derived surface albedo

$\alpha_s(L)$ = surface albedo at $T = T(L)$

$\alpha_s = 0.2$

$\alpha_s(H)$ = surface albedo at $T = T(H)$

$\alpha_s = 0.6$

T = station temperature (°C)

T(L) = station temperature (°C) below which surface albedo is constant (varies with station)

T(H) = station temperature (°C) above which surface albedo is constant (varies with station)

(ii) Anisotropic Model

The calculation of the shortwave radiation incident on an inclined surface, given the intensities of the direct and diffuse shortwave radiation on a horizontal surface, is essentially a problem in trigonometry. The shortwave radiation incident on an inclined surface ($K_{s\downarrow}$) is the sum of the intensities integrated over the full range of local zenith and azimuth angles of the slope in question. Since the radiance distribution for the sky hemisphere is unknown, such a mathematical integration is impossible and, in the case of the diffuse terms, requires some simplifying assumptions.

Calculation of the Direct Shortwave Radiation ($S_{s\downarrow}$)

The formula for calculating the direct shortwave radiation on an inclined surface is not based on any assumptions and thus, for any instant, is exact. The required equations are:

$$S_{s\downarrow} = \frac{S_h \cos i}{\cos \theta}$$

where

$$\cos i = \cos s \cos \theta + \sin s \sin \theta \cos (a-b)$$

$$\cos a = \frac{\sin \phi \cos \theta - \sin \delta}{\cos \phi \sin \theta}$$

and

$$\cos \theta = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H$$

where i is the incidence angle of direct beam irradiance on sloping surface, s is the slope angle, θ is the solar zenith angle, a is the solar azimuth angle from south, b is the sloping surface azimuth angle from south, ϕ is the station latitude, δ is the solar declination and H is the solar hour angle. It should be noted that in these equations both the slope and solar azimuths (aspects) are measured from south, with negative azimuths being toward the east and positive toward the west.

Calculation of the Reflected Shortwave

Radiation $R_s \downarrow$

This term was calculated by invoking the assumptions that adjacent surfaces are horizontal and that they are Lambertian reflectors. This allows the use of the following equation.

$$R_s \downarrow = 0.5 K_h \downarrow \alpha_s (1.0 - \cos s)$$

Calculation of the Diffuse Shortwave

Radiation $D_s \downarrow$

The greatest difficulty in determining the amount of radiation incident on an inclined surface results from the complex distribution of diffuse shortwave radiation over the sky hemisphere, particularly under partly cloudy situations (McArthur and Hay, 1978). The problem has frequently been simplified by invoking an assumed distribution for the diffuse radiance.

Observational (eg. McArthur and Hay, 1978) and numerical studies (eg. Tanaka, 1971) have shown that the degree of anisotropy in the diffuse radiance varies over time. The prime control is the cloud amount, distribution and other physical characteristics. The degree of anisotropy can vary from strongly directional (in the case of a clear atmosphere and small solar zenith angle) to isotropy under a thick and complete cloud cover. (Hay (1978) has argued that hourly integrated values of the atmospheric transmissivity for direct shortwave radiation provide a convenient index (the "anisotropy index") for this variability from strong directionality to isotropy for the diffuse radiance. The anisotropy index for a specific slope is given by:

$$K_s = [I(n) \cos i] / I(0)$$

Since, for a horizontal surface, $\cos i = \cos \theta$, the anisotropy index for a horizontal surface is given by:

$$K_h = [I(0) \cos \theta] / I(0)$$

Thus, for any slope the diffuse radiation to be treated as circumsolar $D_s \downarrow$ is

$$D_s \downarrow = [I(n) \cos i D_h \downarrow] / [I(0) \cos \theta]$$

$$= K_s D_h \downarrow / \cos \theta$$

and the isotropically distributed component

$D_s \downarrow^*$ is

$$D_s \downarrow^* = D_h \downarrow (1.0 - K_h / \cos \theta)$$

With the anisotropic slope radiation model of Hay (1978) the diffuse radiation on the slope is given by:

$$D_s \downarrow = D_h \downarrow [(I(n) \cos i) / (I(0) \cos \theta) +$$

$$0.5(1.0 - I_n / I(0))(1.0 + \cos s)]$$

where $D_h \downarrow$ is the diffuse irradiance on a horizontal surface.

Data requirements:

(i) MAC 3 Model

Along with the values listed in the parameterization section meteorological inputs to the model are the cloud layer amounts and types up to four layers, the total cloud opacity, the dew point temperature, the dry bulb temperature and the station pressure. All these values are required on an hourly basis.

(ii) Anisotropic model

The inputs required for the anisotropic model are the hourly components of the solar irradiance on a horizontal surface which can be either measured or derived.

Accuracy:

The MAC 3 model has been extensively tested by comparing observed vs simulated values at a number of Canadian locations (Vancouver, B.C.; Winnipeg, MAN.; Toronto, ONT.; Montreal, QUE., Charlottetown, P.E.I.; Goose Bay, NFLD.). The mean bias error between observed vs predicted were in most cases within $\pm 1.0\%$ with only one station having a value greater than $\pm 1.0\%$ (-3.3%).

The anisotropic model has been tested by comparing observed vs simulated values at Toronto, ONT. and Vancouver, B.C. The mean error depending on the slope and azimuth direction are mostly less than 10% with many less than 5%.

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CHAPTER 3

MODEL VALIDATION METHODS

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3.1 Introduction

The preceding chapters have provided a compilation of some of the models currently available for the estimation of solar radiation. Models are inherently imperfect estimators and their output, when compared with measured values, are associated with error terms, ϵ_i . Mathematically, the relationship between a model generated value and a coincident measured value may be represented by:

$$y_i = \alpha + \beta x_i + e_i \quad 3.1$$

where y_i is the estimated value,
 x_i is the measured value,
 e_i is the deviation from the best fit line, not to be confused with ϵ_i , the deviation of the generated value from the measured value, and
 α and β are constants defining the best fit line (see Figure 3.1)

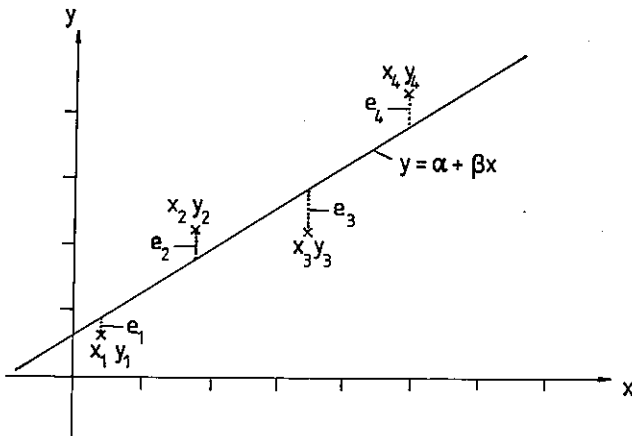


Figure 3.1: Best Fit Curve for Model Generated Values, y_i , vs. Measured Values, x_i

In a perfect estimator, $\alpha = 0$, $\beta = 1$ and $e_i = 0$. As stated above however, models are imperfect and e_i are independent random variables normally distributed about a mean = 0 with a variance = σ^2 .

This error or deviation term may be regarded as the sum of two general components:

(1) Measurement errors. These errors may occur in the measurement of data used as

input to the models or they may occur in the measurement of solar radiation used in model development or both. Latimer (1972) determined, for example, that a root mean squared error (RMSE) of 4% to 5% for both global and diffuse radiation occurs in the Canadian network. These values of error are typical for standard radiation measuring networks. Measurement errors also occur in meteorological and other physical data used as input to most models. These errors may be due to inaccuracies arising from the subjective nature of cloud measurements, from the averaging of parameters not normally measured such as aerosol concentrations, or from instrument and/or reporting errors in other parameters. Because most meteorological parameters are obtained instantaneously at discrete times and solar radiation is normally an integrated amount over a period of time, errors arise in determining representative values of meteorological parameters over the period of integration. To further complicate the process, radiation measurements are based on local apparent time while meteorological observations are based on local standard time.

(2) Stochastic errors. These errors occur because of the inherent irreproducibility of natural phenomena. Even with no measurement error, identical meteorological and physical conditions, over a period of time, will yield different values of solar radiation. These differences are unpredictable and are called stochastic differences. They may also be regarded as the influence of omitted variables in the estimation, each with an individually small effect.

Consequently, certain models may perform better under certain conditions over certain geographic areas than others. While performance or accuracy may be a major criterion in the selection of the most suitable model for a specific application, it is by no means the only factor to be considered. Availability of input data would surely be another major factor. Others may include area of applicability and computational complexity, as a result of which the model could require anything from a large mainframe computer with an extensive memory and rapid processing speed to a small hand-held calculator. Although these factors must all play a role in the selection process, this chapter deals exclusively with the accuracy or performance factor

through the use of statistical comparisons. While it is not the intention of this chapter to provide a theoretical treatment of statistics, some basic background in statistical techniques will be discussed to provide the foundation for the various procedures used in the validation of models. The results of statistical tests of some models are presented as appendices.

3.2 Basic Concepts

The basic premise of model validation is to determine how close the model output values are to coincident measured values. In order to determine the acceptance of a model, however, a reference level must first be established. This reference level may be arbitrarily determined, or it may be pre-governed by other factors such as end-use, or it may be variable as in the case of model comparisons where the model with the greatest accuracy is accepted regardless, within reason, of absolute accuracy.

Once the technique to establish a reference level has been determined, a validation data set must be gathered from which the output from the model or models is to be compared. This data set or test sample must be chosen carefully to avoid factors which may result in misleading conclusions. For example, if the model to be tested were to be used for estimating radiation over all seasons of the year, it may be misleading to test using only data collected during the summer months. Similarly, the use of data which were initially used to develop the model in question would surely result in biased results and should be avoided unless the intent of the model is only to reproduce those data. Aside from these "controllable" influences, there are also chance influences which may bias results. If, for example, in a small random sample, only data from clear days were, by chance selected, the results would clearly bias the performance of models which performed well or poorly under those specific conditions. Although statistical inference of population behaviour through sampling will always introduce uncertainty,

the uncertainty will decrease as the sample size increases, assuming that the sampling is randomly conducted. The validation data sample must, therefore, contain observations which are randomly selected, independent and drawn from the total population and the sample must be sufficiently large to reduce the probability of chance bias. In addition, for the comparison of two or more models, the same validation data must be used in the testing of each model to ensure computational consistency. The succeeding sections will describe a few statistical techniques commonly used in the validation of models.

3.3 Bias Error

In the Introduction to this chapter, a general relationship between a model generated value and a measured value was described in terms of a best fit curve. This curve provides an intuitive description of the performance of the model. It is further exemplified if the unbiased estimator curve, that is, $\alpha = 0$ and $\beta = 1$, were superimposed (see Figure 3.2).

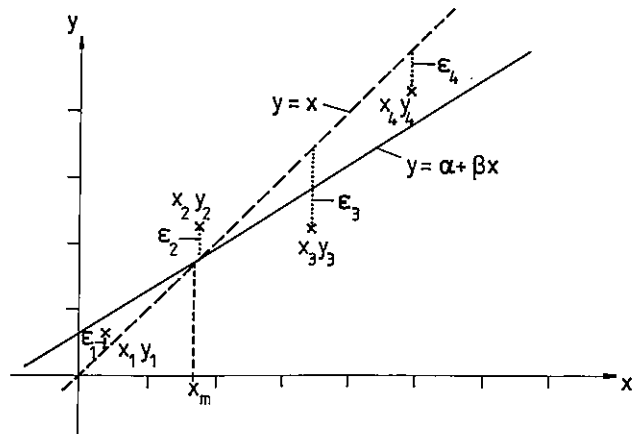


Figure 3.2: Best Fit Curve with Superimposed Perfect Estimator Curve

The closer the best fit curve lies to the unbiased estimator curve the more unbiased the model. For example, in Figure 3.2, for values of x greater than x_m , the intersect point on x of the two curves, this model tends to under-estimate values over the long term; for values of x less than x_m , the model overestimates. Although the deviation term e_i gives a measure of dispersion about the best fit curve, it does not provide a measure of the true deviation from the measured value. The true deviation term ϵ_i shown in Figure 3.2 is, in fact, an error term which includes measurement errors and stochastic errors as well as an error component resulting from the inherent characteristics of the model. In other words, if a model were to contain no inherent systematic inaccuracies or bias, $e_i = \epsilon_i$. Generally,

$$y_i - x_i = \epsilon_i \quad 3.2$$

An analysis, therefore, of the true error term will provide an indication of the bias of the model. Davies (1981) and Page (1979) both used a variant of this term to evaluate models although Davies uses other statistics, which will be discussed later, as well. In both cases a mean value over a specified period was used. Davies refers to this statistic as a Mean Bias Error (MBE) and defines it

$$MBE = \frac{1}{N} \sum_{i=1}^N (y_i - x_i) \quad 3.3$$

where N is the number of observations in the verification data set, and y_i and x_i are the i th calculated and measured values of radiation, respectively (Davies, 1981).

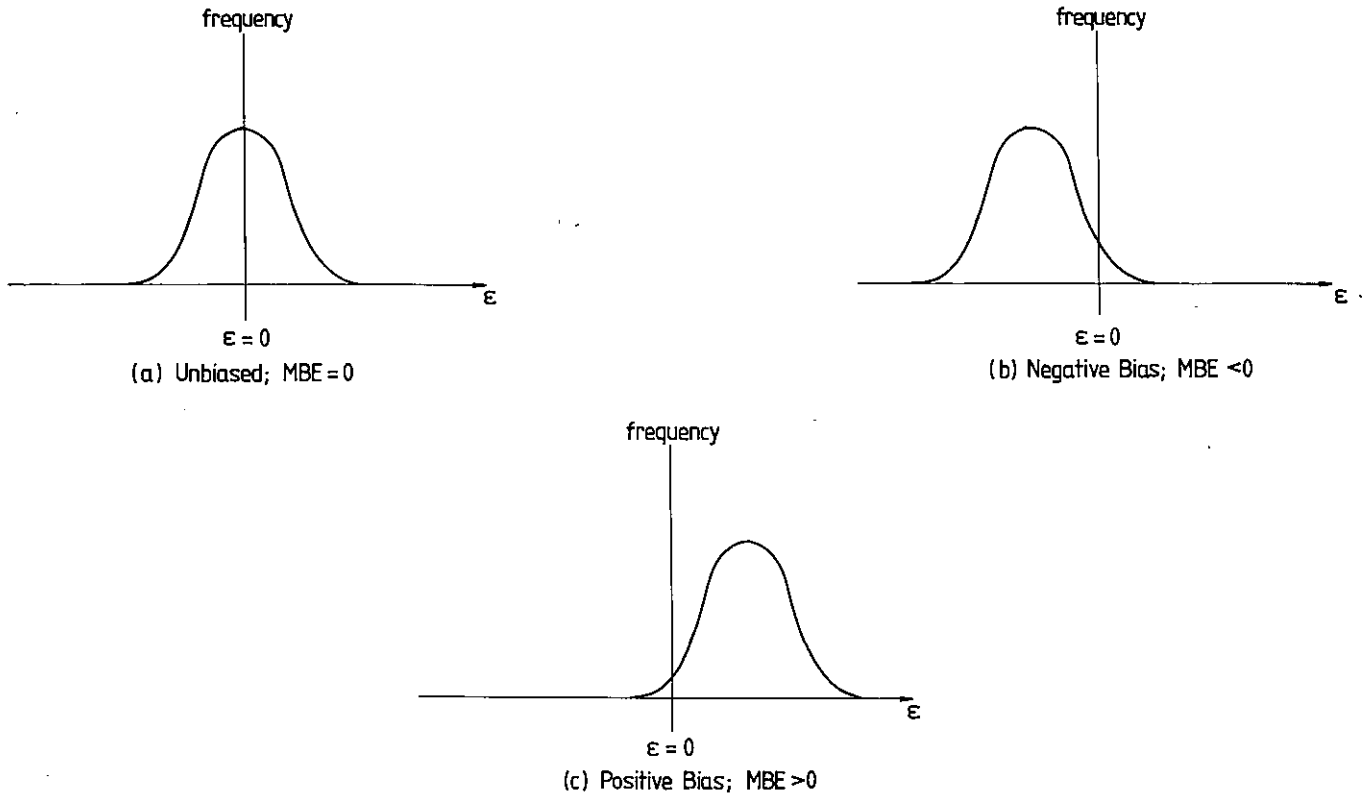


Figure 3.3: Graphs of Frequency Distribution of Deviation or Error ϵ Illustrating Bias

Page (1979) defines the term as a percentage of the measured value, i.e.,

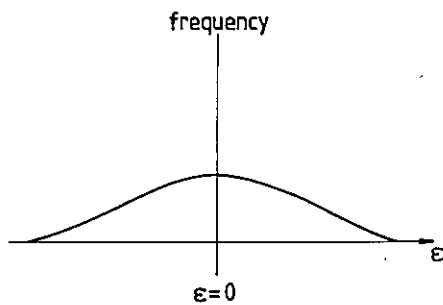
$$\text{ERROR} = \left(\frac{\text{Calculated}}{\text{Measured}} - 1 \right) \times 100\% \quad 3.4$$

Both equations 3.3 and 3.4 provide an indication of the bias; positive values indicate over-estimation while negative values show an under-estimation of the generated value. Davies uses hourly and daily totals in his assessments over individual years, all years and seasonally, and Page compares monthly mean daily totals then computes the annual mean.

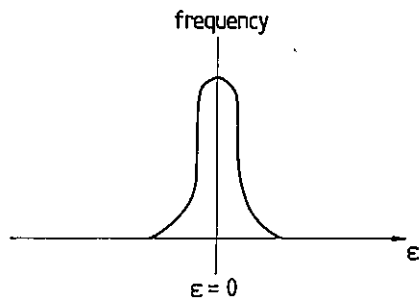
Graphically, a biased and an unbiased model may be depicted in terms of the true deviation term ϵ distribution as shown in Figures 3.3 (a), (b) and (c).

3.4 Variance of Errors

While the degree of bias of a model is an extremely useful statistic, it does not necessarily provide any indication of its accuracy or its predictability. For example, suppose Figures 3.4 (a) and (b) illustrate the error distributions of two unbiased models.



(a) Inefficient Estimator; large s^2



(b) Efficient Estimator; small s^2

Figure 3.4: Graphs of Frequency Distributions of Deviation or Error ϵ Illustrating Efficiency

It is clear that the model whose error distribution as shown in Figure 3.4 (b) would provide a more accurate value over the long term. A measure of this dispersion characteristic is the variance defined by:

$$s^2 = \frac{1}{N-1} \sum_{i=1}^N (\epsilon_i - \bar{\epsilon})^2 \quad 3.5$$

$$\text{where } \bar{\epsilon} = \frac{\sum_{i=1}^N \epsilon_i}{N}$$

For an unbiased estimator,

$$\bar{\epsilon} = 0 \quad 3.6$$

Therefore, a biased model which exhibits stronger variance characteristics, that is, a lower variance, may be more desirable than an unbiased model with a wide error dispersion. A wide error dispersion characteristic may give an indication of the unpredictability of a model.

The variance of the error, as discussed, is a measure of its variation about its mean. By itself, it does not infer any indication of the true accuracy of the model. Together with the bias error, some measure of accuracy is achieved. A widely used statistic, the root mean squared error, provides the gauge for determining the closeness of the model generated value to the coincident measured value.

3.5 Root Mean Squared Error

As discussed in previous sections, no one statistic provides a complete picture of the performance of a model. The bias error gives the tendency of a model, over the long term; the error variance gives the dispersion of errors about its mean value. The root mean squared error (RMSE) provides a term by term comparison of the actual deviation between calculated value and measured value and is defined:

$$\text{RMSE} = \left(\frac{\sum_{i=1}^N \epsilon_i^2}{N} \right)^{\frac{1}{2}} \quad 3.7$$

or

$$\text{Mean Squared Error} = \frac{\sum_{i=1}^N \epsilon_i^2}{N} \quad 3.8$$

For an unbiased model, equations 3.5 and 3.8 are similar and converge as $N \rightarrow \infty$, however, most models exhibit some bias and the RMSE will provide a truer picture of its deviation from observed values. Because of the squared term, positive and negative errors do not cancel, but a few large terms will amplify the final result. Nevertheless, this statistic has been widely used in the validation of models, (Davies, 1981; Won, 1977; Suckling and Hay, 1976; etc.). In order to standardize relative comparisons between different data sets, it is often useful to display RMSE in terms of a percentage of the measured value.

3.6 Correlation

While the aforementioned statistics are necessary in the total evaluation of models they do not specifically provide an indication of the behaviour of the model-generated values with respect to their respective observed values. A measure of the degree of linear relationship between the two variables, that is, between calculated values and measured values, is their correlation coefficient defined as:

$$r_{xy} = \frac{1}{N-1} \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{s_x s_y} \quad 3.9$$

and since

$$s_z = \left(\frac{1}{N-1} \sum_{i=1}^N (z_i - \bar{z})^2 \right)^{\frac{1}{2}}$$

equation 3.9 becomes

$$r_{xy} = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{\left(\sum_{i=1}^N (x_i - \bar{x})^2 \right)^{\frac{1}{2}} \left(\sum_{i=1}^N (y_i - \bar{y})^2 \right)^{\frac{1}{2}}} \quad 3.10$$

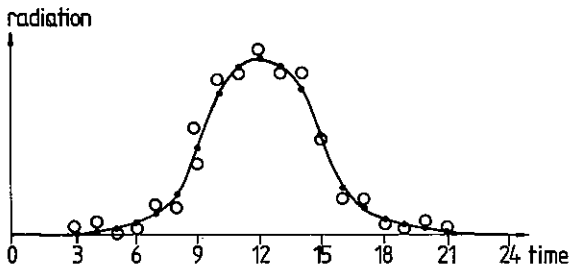
An exact linear relationship, that is, $x_i = y_i$, would yield a correlation coefficient of 1. Because of the axis translation resulting from the movement of the origin to (\bar{x}, \bar{y}) , the units of measurement do not play a role in the correlation. To achieve a high value, that is, near 1, a negative x value, with respect to (\bar{x}, \bar{y}) must be accompanied by a corresponding negative y value. Similarly, positive x's are accompanied by positive y's.

As in the other statistics, the correlation coefficient by itself, although a useful and informative statistic, will not provide a complete picture of the accuracy of a model but must be used in conjunction with other statistics.

3.7 Analysis of Statistics

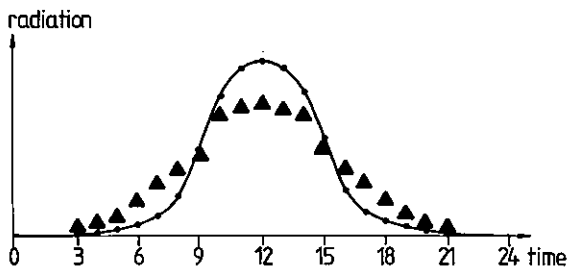
It is clear from the preceding discussions that no one statistic can provide a complete picture of the performance of a model, in fact, use of only one statistic may be misleading in evaluating total model performance. For example, the mean bias error statistic of a model may reveal a very low value indicating, perhaps, a promising method to consider. Further investigation, however, may reveal that in the computation of the MBE, gross overestimates were offset by equally gross under-estimates. In all cases, the eventual application of the model should be considered in determining a valid data set for verification. As most models exhibit some degree of bias dependent on geographical, seasonal and diurnal variations or, perhaps, radiation intensity, these factors should also be considered in data selection for validation. If, for example, a model is to be used for radiation estimation only during the winter months, validation data should be chosen from those months. While the test results may not be representative for other seasons, they will be valid for the intended use.

Generally, model validations are carried out with no specific application in mind but to test the acceptability and perhaps the versatility of the methods. In such tests, the factors mentioned in the preceding paragraph must also be evaluated. A validation data set must be sufficiently large and span seasonal variations and different geographical locations. For completeness, the validations should also be carried out seasonally to detect seasonal biases and, in the case of hourly radiation, on an hour by hour basis to verify diurnal tendencies. A time series analysis of errors will provide the detail in model evaluation which may be masked in the statistical computations. Figures 3.5, 3.6 and 3.7 graphically illustrate some error types pertaining to the performance of a single model.



• Measured
○ Calculated

Figure 3.5
Graph of Measured Daily Radiation with Model
Generated Values Showing Ideal Error Shape
(After MacLaren, et. al., 1979)



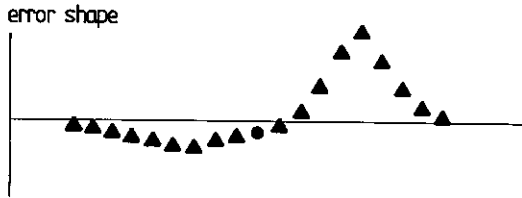
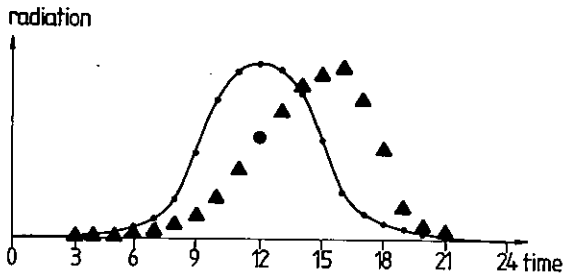
• Measured
▲ Calculated

Figure 3.6
Graph of Measured Daily Radiation With Model
Generated Values Showing Symmetrical Diurnal
Bias (After MacLaren, et al., 1979)

It was stated in Section 3.2 that a reference level must be established before determining the acceptability of a tested model. It can be seen that this reference level is, in fact, a number of criteria combined to result in an overall assessment. These criteria may be arbitrarily assigned, such as $|MBE| < 5\%$ and a $RMSE < 20\%$ or they may be predetermined by the end-use; for example, for the design of solar collector facilities where perhaps the limits may be set to $|MBE| < 5\%$ and the extreme $RMSE < 40\%$

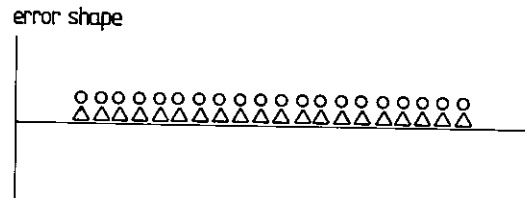
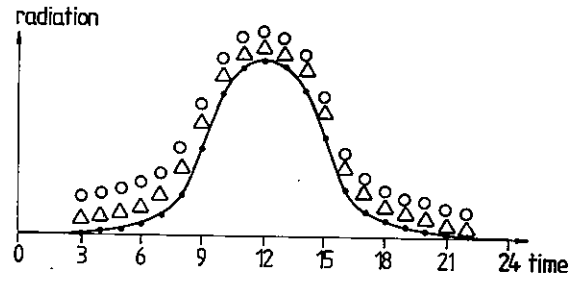
In the comparison of models, it is essential that the identical verification data set is used for all models. Care should also be taken to ensure that the data set is sufficiently large so that there is no inherent bias in the procedure. As it is with the validation of one model, end-use should be a factor to be considered when comparing the performance of more than one. An end-use may be the general versatility of models, in which case, a series of validation tests are necessary to evaluate their performance properly. These tests, similar to the single model evaluations, may be used to detect seasonal, geographical or diurnal biases. It is also informative to compute validation statistics over various summation periods beginning with the finest resolution for which the model(s) were intended. For example, statistics for hourly radiation models should be computed for hourly data as well as for daily, weekly, monthly and perhaps annual sums. MacLaren, et al. (1979) found in an evaluation of Canadian models that typical RMS errors for hourly global irradiance were of the order of 25% and for daily global irradiance, typical errors were 15%. In general, increasingly smaller errors may be expected for increasing summation periods.

It was mentioned earlier, in the comparison of models, that the same validation data sets should be used. While this may not be possible in many cases due to variations in input parameters, data from the same locations and for the same period of record should be chosen. With a sufficiently large data set, and neglecting differences in data reporting, that is, at any one location relative reporting accuracies are constant for all data, the comparison should be valid.



• Measured
▲ Calculated

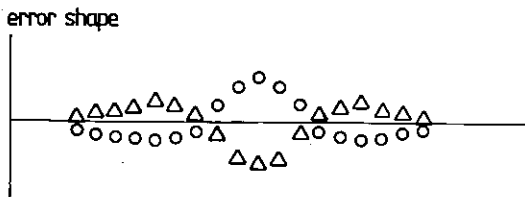
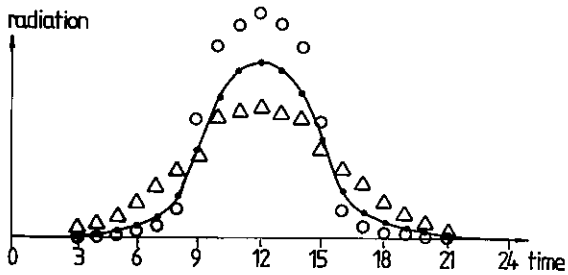
Figure 3.7
Graph of Measured Daily Radiation with Model Generated Values Showing Skewed Diurnal Bias (After MacLaren, et al., 1979)



• Measured
Calculated
△ Model A
○ Model B

Figure 3.8
Graph of Measured Daily Radiation with Generated Values from Two Models; Symmetrical, Positive Bias (After MacLaren, et al., 1979)

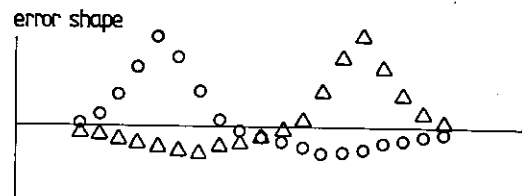
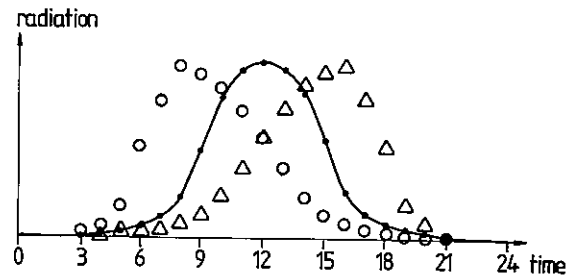
$$\begin{aligned} \text{MBE} (\circ) &> \text{MBE} (\triangle) \\ \text{RMSE} (\circ) &> \text{RMSE} (\triangle) \\ r (\circ) &= (\triangle) = 1 \end{aligned}$$



• Measured
Calculated
△ Model A
○ Model B

Figure 3.9
Graph of Measured Daily Radiation with Generated Values from Two Models; Symmetrical, Offsetting Bias (After MacLaren, et al., 1979)

$$\begin{aligned} \text{MBE} (\circ) &= \text{MBE} (\triangle) = 0 \\ \text{RMSE} (\circ) &= \text{RMSE} (\triangle) \\ R (\circ) &= r (\triangle) \end{aligned}$$



• Measured
Calculated
△ Model A
○ Model B

Figure 3.10
Graph of Measured Daily Radiation with Generated Values from Two Models; Skewed, Offsetting Bias (After MacLaren, et al., 1979)

$$\begin{aligned} \text{MBE} (\circ) &= \text{MBE} (\triangle) = 0 \\ \text{RMSE} (\circ) &= \text{RMSE} (\triangle) \\ r (\circ) &= r (\triangle) \end{aligned}$$

While the computation of mean statistics may provide a clear indication of model performance, there are cases when further detail is required for a true assessment of model use. As indicated for the evaluation of single models, time series analyses of errors may provide the detail necessary to determine model validity. Analogous to Figures 3.5, 3.6 and 3.7, Figures 3.8, 3.9 and 3.10 illustrate graphically, error analyses of two models.

While it is clear both from the statistical and the graphical points of view that in Figure 3.8, model A gives a better performance, the statistics and the error analyses as shown in Figures 3.9 and 3.10 are not so straightforward. Although from a statistical aspect, the two models in each case are identical, a marked difference appears when a time series of their respective errors is analysed. A similar analysis may be conducted on a seasonal basis as well. Such analyses may provide very useful information in evaluation model end-use.

3.8 Conclusions

It has been ascertained that no single statistic can provide a true indication of model performance or accuracy. It is, in fact, a combination of statistical evaluations and a knowledge of end-use or application which determine the acceptability of a model. This end-use or application, which may be a general indication of versatility and acceptability, determines, in part, a reference acceptance level by which models may be evaluated.

In the estimation of solar radiation, as it is in the estimation of all natural phenomena, there are inaccuracies caused by measurement errors as well as stochastic errors. In addition, there may be an error component resulting from the inherent characteristics of the estimation method or the model. The validation process, which evaluates these inaccuracies, uses a measured data set which is considered the "standard" or reference values. Care must be exercised in the selection of validation data sets to avoid accidental or "chance" biases. Some of the major determinants of bias are due to geographical, seasonal, diurnal and meteorological effects. To reduce the probability of chance bias, the validation data set should:

- i) be randomly selected;
- ii) be independent of the model(s) being evaluated;
- iii) span all seasons;
- iv) be selected from various geographical regions; and
- v) be sufficiently large to include a spectrum of weather conditions.

To ensure computational consistency and output comparability between models in an inter-model comparison, validation data for all models should be from the same data set.

In the computation of statistics in the validation of models, one of the most widely used parameters to indicate accuracy is the root mean squared error (RMSE). It provides a measure of the deviation of the model generated value from the measured value. While the RMSE indicates degree of departure of model produced values from observed, it does not provide any indication of the tendency or bias of the model; that is, whether the model over-estimates or under-estimates, etc., over the long term. The mean bias error (MBE) or mean error will measure this tendency. A zero MBE indicates as much negative deviation as positive over a long term. Together with the RMSE, the MBE provide a reasonably comprehensive evaluation of model performance. Other statistics such as: correlation coefficient, error variance, skew, etc., may also be computed to provide greater definition of performance.

To provide a clearer indication of the versatility and applicability of a model, it is advisable to compute the validation statistics on a monthly or at least a seasonal basis as well as on an annual basis. Seasonal variations in performance can then be readily detected. An analysis of hour by hour statistics, for hourly radiation models, would complete the evaluation providing an indication of diurnal tendencies.

Statistics can be a very useful tool or it can be a hindrance in the validation of radiation models. If used in a knowledgeable and objective manner it can provide a numerical indication of the performance of a model. It is important, therefore, to understand both the strengths as well as the weaknesses of any statistic used.

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CHAPTER 4

WEATHER DATA SETS FOR VALIDATION
OF INSOLATION ALGORITHMS

by

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WEATHER DATA SETS FOR VALIDATION OF INSOLATION ALGORITHMS

Earlier work by various researchers has shown that algorithms for the conversion of measured global radiation on horizontal surface or measured sunshine duration, to radiation on inclined surfaces, gave reasonable results only over longer periods, 10 days or more, whereas the conversion of daily or hourly values often contained rather large deviations from measurements.

To enable workers in the IEA countries to test algorithms for the calculation of solar radiation on inclined surfaces using weather data for a variety of climates it was decided in the task 5 group to establish a computer tape with a library of weather data sets. These data sets should contain both measured solar irradiation on inclined surfaces and such other weather data from which this radiation would be computed by various algorithms.

The background for this decision was that many of the known insolation models (algorithms) for radiation on inclined surfaces had been developed and tested with data from only one location, and sometimes for rather short periods of measurements.

The Thermal Insulation Laboratory, Technical University of Denmark therefore undertook the work of compiling and distributing a computer tape with hourly values for a number of locations.

A request to meteorological services in the participating countries, and to other involved researchers, asked for weather data and for permission to distribute these data for the above mentioned purpose.

European data

Tapes with hourly data were received from 11 locations in Europe. Two of these data sets were tagged with stricter limitations for the use and distribution than the others, and were therefore omitted from the final computer tape.

The remaining data sets for 9 locations contain as a minimum hourly measured values of global radiation, diffuse radiation and

either radiation on one or more inclined surfaces, or direct normal radiation. Most of the data sets include also sunshine duration, daily or hourly, and a few of them other weather parameters also. (Appendix F).

To facilitate the use of these data they have all been brought to a common format and common units. (Appendix G). The data are written on a computer tape (name VIA 1). Every effort has been made to make it useable on as many computer installations as possible.

Availability

The data will be made available for scientific purposes. The tape is obtainable from
Thermal Insulation Laboratory
Technical University of Denmark
DD-2800 Lyngby, Denmark.
(Price: 500 Danish Crowns).

US data

For a large number of stations SOLMET tapes are available, containing hourly global radiation and collateral surface meteorological data for many years.

For a number of these stations (in 1978 = 27 stations) the global radiation has been rehabilitated, corresponding to the present state-of-the art.

The SOLMET-format contains fields for direct normal radiation (field 102) and global radiation on a tilted surface (field 105). However, these data are generally computed values, and therefore not suitable for the purpose here.

Canadian data

Two Canadian stations have measured radiation on inclined surface for some years, Vancouver (U.B.C., 77.01 - 78.12) and Toronto (M.R.S., 76.12 - 78.03).

The data included are global, diffuse and direct normal radiation, and global radiation on vertical and 30° and 60° tilted south facing surfaces.

For 50 stations tapes are made with computed radiation data and measured meteorological data.

Acknowledgements

The following institutions or organisations have kindly delivered the weather data for these data tapes:

Centre National de la Recherche Scientifique
Laboratoire d'Energétique Solaire
BP n^o 5, Odeillo, F-66120 Font-Romeu, France

Irish Meteorological Service
44, Upper O'Connell Str.
Dublin 1, Ireland

Swedish Meteorological and Hydrological
Institute
S-601 76 Norrköping, Sweden

Meteorological Office
Eastern Road, Bracknell, Berkshire
RG 12 1UR, United Kingdom

Swiss Meteorological Institute
Krähhühistr. 58
CH-8044 Zürich, Switzerland

Joint Research Centre, Ispra
I-21010 Ispra (Varese), Italy

Danish Meteorological Institute
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DK-2100 Copenhagen Ø, Denmark

Hydrotechnical Laboratory
The Royal Veterinary and Agricultural High
School
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APPENDIX A (CHAPTER 1)

LIST OF SYMBOLS WITH SIMILAR DEFINITIONS.

Bener	Definition of Symbol	Davies
c	total cloud amount	CA
H ₂	diffuse irradiance	D
H ₁	cloudless sky diffuse irradiance	D ₀
G ₂	global irradiance	G
G ₀	daily total extraterrestrial irradiance	G(0)
G ₁	global irradiance for cloudless skies	G ₀
τ	hour angle	H
S ₀	solar constant (spectrally integrated direct beam irradiance at the top of the atmosphere normal to the Sun) corrected for the instantaneous departure of the actual Sun-Earth distance from the mean value	I(0)
S ₀	extraterrestrial irradiance	I(0)
S ₁	cloudless sky direct beam irradiance	I ₀
ρ ₂	actual Sun-Earth distance	R*
q _a	transmittance after extinction by aerosols	T _r (a)
q _o	transmittance after absorption by carbon dioxide and ozone	T _r (o)
q _R	transmittance after scattering by dry air molecules	T _r (R)
w	transmittance after absorption by water vapour	T _r (w)
m	relative optical air mass	m _r
b	station pressure	p
σ	hourly fraction of bright sunshine	s
δ	solar declination	δ
Z	zenith angle	θ
λ	wavelength	λ
m ^k _{pλ}	spectral optical depth due to absorption and scattering (extinction) by particulates	τ _{aλ}

$m (k_{a\lambda} - k_{w\lambda})$	spectral optical depth due to absorption by gases	$\tau_{g\lambda}$
$mk_{R\lambda}$	spectral optical depth due to molecular (Rayleigh) scattering	$\tau_{R\lambda}$
$mk_{w\lambda}$	spectral optical depth due to absorption by water vapour	$\tau_{w\lambda}$
a	azimuth angle	ϕ
ϕ	station latitude	ξ

APPENDIX B (CHAPTER 1)

SURVEY AND COMMENTS ON VARIOUS METHODS TO COMPUTE THE COMPONENTS
OF SOLAR IRRADIANCE ON HORIZONTAL AND INCLINED SURFACES

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LIST OF SYMBOLS

a	Solar azimuth, also arbitrary constant	k_λ	Total spectral extinction coefficient
b	Barometric pressure, also arbitrary constant	$k_{a\lambda}$	Spectral absorption coefficient relating to the absorption by all atmospheric gases
c	Cloudiness in tenths, also arbitrary constant	$k_{o\lambda}$	Spectral ozone absorption coefficient. Similarly the symbol λ in the indices of the other coefficients designates the spectral value of the coefficient.
d	Arbitrary constant	$k_{o\lambda l}$	Spectral ozone absorption coefficient for an amount of $x = 1.0$ cm NTP of ozone. Similarly the symbol l in the indices of other coefficients designates normalized values relating to unit values or otherwise stated values of the relevant parameters
e	Arbitrary constant	m	relative airmass according to Bemporah relating to the lightpath of direct solar radiation. In many cases the approximation $m = \sec z$ is used for not too large values of the zenith angle z
f	Arbitrary constant	m_ζ	Relative airmass relating to the zenith angle ζ of an arbitrary celestial point. Approximately $m = \sec \zeta$.
f(x)	Arbitrary function	n	Index of summation or enumeration respectively.
g	Arbitrary constant	o	Index, designating extraterrestrial values of solar or global intensity. Designates furthermore "Ozone" in k_o .
h	Solar altitude	p	Designates as an index in k_p particle scattering.
h'	Angular altitude of a celestial point	q	Total integral transmission factor of the atmosphere for a relative airmass $m = 1$
i	Enumeration index	q_a	Integral transmission factor relating to the absorption by all atmospheric gases. The indices applied for marking the different kind of atmospheric transmission are the same as those used in connection with the coefficients k
j	Relative sky radiance distribution with respect to zenith radiance		
k	Total integral extinction coefficient of the atmosphere relating to airmass $m = 1$		
k_a	Integral absorption coefficient relating to the absorption by all atmospheric gases		
k_o	Integral ozone absorption coefficient		
k_{ps}	Particle scattering coefficient		
k_{pa}	Particle absorption coefficient		
k_p	Particle extinction coefficient = $k_{ps} + k_{pa}$		
k_R	Integral Rayleigh scattering coefficient		
k_w	Integral absorption coefficient for water vapour		
k'	Pseudoturbidity coefficient according to Unsworth		

s	Angle of inclination of a surface with respect to the horizontal. As index, s designates "scattering"		Indices 1 or 2 attached to H designate unclouded or clouded conditions respectively. As an index, H refers to sky radiation.
t	Designates time or instantaneous values respectively	I	Designates any of the intensities S, H, G, Z, radiance J or relative radiance j.
At	Time interval	\hat{I}	The intensities etc. represented by I taken in their photometric significance and expressed in photometric units.
w	Precipitable water expressed in cm NTP	J	Sky radiance (in $\text{Wm}^{-2}\text{ster}^{-1}$)
x	Total amount of ozone expressed in cm NTP	\hat{J}	Sky luminance (in cd m^{-2})
z	Zenith angle of the sun	K_I	Luminous efficiency in lmW^{-1} relating to any of the radiometric quantities S, H, J etc. represented by I.
A	Albedo	K'	Equals K_z/K_J , explained in text, section 8.2.
B	"Background sky radiation", assumed as isotropically distributed	M	Refers as an index to measured values, e.g. H_{2M}
C_i	Correction factors according to Temps & Coulson (50) considering the influence of anisotropy of sky and ground reflected radiation ($i=1-3$). Valid for unclouded sky. Symbols C_i serve also as constants	P_s	Ratio between direct solar irradiance or irradiation on inclined and horizontal surfaces respectively.
C'_i	Correction factors similar to C_i , but generalized by Klucher (58) to include also clouded sky conditions ($i=1,2$)	P_H	Same for sky irradiance or irradiation
D	Diffuse radiation including sky radiation and ground reflected radiation	P_G	Same for global irradiance or irradiation
F	Designates factors explained in text	P_R	Ratio between irradiance by ground reflected radiation on an inclined surface and global irradiance on a horizontal surface
G	Global radiation on a horizontal or inclined surface. Indices 0,1 and 2 relate to extraterrestrial values, unclouded and clouded conditions respectively. As an index, G refers to global radiation	Q	Characteristic ratio of two radiation quantities or components such as e.g. G_2/G_0 or G_1/S_0
\hat{G}	Global illumination in lx or lm m^{-2} . A symbol marked means that the radiation quantity in question is taken in its photometric significance and expressed in photometric units	R	Intensity of ground reflected radiation
H	Sky radiation on a horizontal or inclined surface. Does not include ground reflected radiation.	S	Direct solar intensity. The indices 0, 1 and 2 refer to extraterrestrial solar radiation, unclouded and clouded conditions respectively

$S_{0\lambda}$	Spectral intensity of extraterrestrial solar intensity	τ_s	Sunset hour angle for a horizontal plane. Further designations relating to sunset and sunrise hour angle for otherwise orientated surfaces are explained in text
S'_0	Designates S_0 corrected for the variable sun-earth distance		
S^*_i	Intensity of direct solar radiation for cloudless conditions reduced by Rayleigh scattering and absorption by atmospheric gases	ϕ	Azimuth angle of a celestial point
$\Delta S'_a$	Amount of solar intensity attenuated by the different components of atmospheric absorption and corrected for variable sun-earth distance	Ψ	Scattering angle, resp. angle between the direction to the sun and the direction to the celestial point considered
S'_a	Solar intensity reduced by absorption, but not by Rayleigh and aerosol scattering	θ	Angle of incidence
		ϕ	Latitude
		Ω	Angular area.
T	Turbidity factor according to Linke		
U	Empirical factor in equation (71)		
$V(\lambda)$	Normalized visual sensitivity function		
$W(n)$	Empirical parameter in equation (23) represented in dependence of the number n of the day by equation (24)		
Z	Intensity of circumsolar radiation (see text, section 9)		
α	Wavelength exponent according to Ångström/Schüepp		
β	Turbidity factor according to Ångström/Schüepp		
γ	Azimuth angle of the inclined surface considered, counted westwards from the southern meridian		
δ	Declination of the sun		
ζ	Zenith angle of a celestial point		
λ	Wavelength		
ρ^2	Factor taking account of the variable sun-earth distance		
σ	Relative sunshine duration		
τ	Hour angle, counted westwards from the southern meridian		

Explanation of the symbols put in brackets

The irradiances discussed in this paper may relate to horizontal or inclined surfaces. Instantaneous, hourly, daily values etc. are considered. The averaging time interval may be an hour, a day, a decade or a longer period. The symbols in the rectangular brackets, attached to the symbols of radiometric quantities, state which of the different cases applies. The designations are as follows:

- N Surface normal to the incoming radiation
- H Surface horizontal
- G Surface inclined, surface azimuth arbitrary
- t Instantaneous values
- h Hourly values
- d Daily values
- 10d Decadic values resp. Averaging period 1 decade
- m Monthly values resp. Averaging period 1 month
- p designates an averaging period in general
- lp designates a very long averaging period
- I[G,h,p] Hourly values of I on an inclined surface, averaged over period p
- G[G,d,m] Daily values of global irradiation G on an inclined surface, averaged over the period of a month
- H[H,d, 10d] Daily values of sky irradiation on a horizontal surface, averaged over the period of a decade
- S[N,t] Instantaneous value of direct solar intensity

- S[H,t] Instantaneous values of direct solar irradiation on a horizontal surface, resp. vertical component of S(N)
- Z[N,t] Instantaneous values of circumsolar radiation (aureole) approximately treated as parallel radiation incoming from the direction of the sun
- Z[H,t] Instantaneous values of circumsolar irradiation on a horizontal surface, assuming the same approximation
- R[H] Diffuse radiation reflected from the horizontal ground
- R[G] Diffuse irradiance by ground reflected radiation on an inclined surface
- σ [d,lp] Daily relative sunshine duration, averaged over a very long period
- P_s [t] Ratio P_s computed from instantaneous values of direct solar irradiance
- P_s [d] Ratio P_s computed from daily values of direct solar irradiation

Remarks: Frequently the significance of the symbols S, H, G etc. are sufficiently clear from the context. The brackets are omitted in these cases. Furthermore, the specification of the averaging period (e.g. month) and of the time interval (e.g. hour) is not always included in the bracket, but mentioned in text.

INTRODUCTION

A large number of various methods to compute solar, sky and global radiation have been developed during the last decades, ranging from Chandrasekhar's theory of radiative transfer to very simple models (Goldberg, Klein and McCartney 40). The present paper gives a systematic survey on the different procedures applied. Mainly simpler models involving more or less elementary mathematics, semiempirical approaches or numerical approximation of statistical results are considered. Computations of sky radiation based on a rigorous solution of the problem of multiple scattering in the atmosphere are shortly mentioned. No all-embracing discussion of existing models and computations could be attempted. So far, the list of publications given in the references is not complete. The surveys by May (67), Perrin de Brichambaut (68) and Krochmann (69) offered most valuable information for the preparation of the present report. It seemed adequate for the sake of a systematic representation of the various methods to discuss the cases of solar, sky and global radiation, as well as of unclouded and clouded sky and of horizontal and inclined receiving surfaces in separate sections. The computing model of one author is therefore treated in different sections. Uniform symbols are applied except when otherwise mentioned.

B 1. COMPUTATION OF DIRECT SOLAR INTENSITY S_1 , NOT INFLUENCED BY THIN OR DENSER LAYERS OF CLOUD

Proceeding from the extraterrestrial spectral intensity $S_{0\lambda}$ and taking account of the attenuation by the various atmospheric components, the integrated solar intensity S_1 can be computed in a straightforward manner according to the law of Bouguer and Lambert. The different coefficients for extinction by atmospheric scattering and absorption depend on wavelength. The molecular scattering and aerosol scattering is governed by the theories of Rayleigh (1) and Mie (2) respectively. A well known and widely applied expression for the wavelength variation of the aerosol scattering coefficient according to formula (6) below has been given by Ångström (3) and Schüepp (4). The data on

atmospheric absorption are based on the results of measurements and theoretical investigations on the structure of the absorption bands of the relevant absorbing gases (O_2 , O_3 , H_2O and CO_2).

In the visible (380 - 780 nm) the influence of absorption (by O_2 and O_3) is small, whereas Rayleigh scattering and Mie extinction are relevant. In this wavelength region the direct solar intensity can be computed from Rayleigh's and Mie's theories. The size distribution and vertical distribution of the aerosol, as well as the complex refractive index of the particles, must be given among others as input data.

B 1.1 Computation of direct solar intensity S_1 from spectral data on the coefficients of atmospheric scattering and absorption

The integrated intensity S_1 of direct shortwave solar radiation is given in good approximation by the expression:

$$S_1 = \rho^2 \int_{\lambda_1}^{\lambda_2} S_{0\lambda} e^{-m(k_{R\lambda} + k_{p\lambda} + k_{a\lambda})} d\lambda \quad (1)$$

Here the interval of integration covers the region between $\lambda_1 = 0.29 \mu\text{m}$ to $\lambda_2 = 3.0 \mu\text{m}$.

The coefficients of extinction relate to a relative air mass $m = 1.0$, resp. to the whole column of the atmosphere above the ground level considered. The coefficient $k_{p\lambda}$ for particle extinction includes the coefficients $k_{p\lambda s}$ and $k_{p\lambda a}$ for particle scattering and particle absorption respectively. The values of these quantities can be determined by means of Mie's theory from the aerosol model assumed. On the other hand the scattering coefficient $k_{p\lambda s}$ may be computed from the empirical formula (6) by Ångström and Schüepp, mentioned further down.

The coefficient $k_{a\lambda}$ includes the absorption coefficients of the atmospheric gases water vapour, ozone, carbon dioxide and oxygen:

$$k_{a\lambda} = k_{w\lambda} + k_{o\lambda} + k_{\lambda} (CO_2) + k_{\lambda} (O_2) \quad (2)$$

Introducing the normalized values $k_{R\lambda 1}$, $k_{w\lambda 1}$, etc. for the attenuation coefficients we have

$$k_{R\lambda} = k_{R\lambda 1} \frac{b}{1000}, \quad (3a)$$

$$k_{\lambda}(\text{CO}_2) = k_{\lambda 1}(\text{CO}_2) \frac{b}{1000}, \quad (3b)$$

$$k_{\lambda}(\text{O}_2) = k_{\lambda 1}(\text{O}_2) \frac{b}{1000} \quad (3c)$$

$$k_{w\lambda} = k_{w\lambda 1} \cdot w \quad (3d)$$

$$k_{o\lambda} = k_{o\lambda 1} \cdot x \quad (3e)$$

where b (mb), w (cm) and x (cm) stand for air pressure at ground level, precipitable water and amount of ozone respectively. Thus, these formulae give the value of the various coefficients in dependence on the values of the parameter relevant for each coefficient. With (2) to (3e) relation (1) becomes:

$$S_1 = \rho^2 \int_{\lambda_1}^{\lambda_2} S_{0\lambda} \exp - m [(k_{R\lambda 1} + k_{\lambda 1}(\text{CO}_2) + k_{\lambda 1}(\text{O}_2)) \frac{b}{1000} + k_{p\lambda} + k_{w\lambda 1} \cdot w + k_{o\lambda 1} \cdot x] d\lambda \quad (4)$$

As an example for this method of computation the calculations of Robinson and Schüepp (5) may be mentioned, whose approach corresponds to the relations given in the formulae (1) resp. (4) above. These computations are based in part on the theoretical formula (5) for the Rayleigh scattering coefficient $k_{R\lambda 1}$ and on the Ångström/Schüepp (4) empirical relation (6) for the particle scattering coefficient $k_{p\lambda s}$.

$$k_{R\lambda 1} = 0.00386\lambda^{-4.05} \quad (5)$$

$$k_{p\lambda s} = \beta(2\lambda)^{-\alpha} \quad (6)$$

Measurements carried out in different climates have shown that a value of $\alpha = 1.5$ is adequate for statistical calculations, so long as only total radiation is considered. The authors used the results of Fowle (6) and of Mörikofer & Schüepp (7) to compute the attenuation by absorption. Values of $m \cdot k_{a\lambda}$ derived from these data are given in a table for different wavelength intervals from $\lambda = 0.72 \mu\text{m}$ to $\lambda > 3.0 \mu\text{m}$ and for values of $m \cdot w$

ranging between $0.1 \leq m \cdot w \leq 30$ cm. These values of $m \cdot k_{a\lambda}$ take account of the absorption by water vapour, carbon dioxide and oxygen, but not, in this case, of ozone absorption. The latter effect, i.e. the term $m \cdot k_{o\lambda 1} \cdot x$, is computed from mean seasonal values of the amount of ozone x , obtained by Goetz et al. (8). The standard values for the parameters x , w and β which the authors used to compute direct solar intensity S_1 are presented in tables.

A more detailed presentation of the method of computation considered above in this section is given, for example, in a CIE Technical Report by Aydinli (9).

The results obtained by means of the expressions (1) or (4) are sufficiently accurate for many purposes, provided that reliable values of the various parameters are available. However, these expressions represent only an approximation: The effect of each component of atmospheric attenuation is taken account of by one single value of extinction coefficient per wavelength interval $\Delta\lambda$. The same value of relative airmass m is applied for the different sources of attenuation. According to a more sophisticated model of computation the atmosphere is broken up in vertical direction into a large number of horizontal layers. The density of the different atmospheric components within the various layers is obtained from the model atmosphere considered. The pathlengths within each layer along the curved lightpath is computed and the extinction produced by the different sources of attenuation determined for small intervals of wavelength. Summing up the contributions of the many layers to the attenuation yields the spectral intensity $S_{1\lambda}$ for the wavelength interval $\Delta\lambda$ considered. Furthermore the total intensity for a given wavelength region λ_1 to λ_2 is calculated by numerical integration. Computer programs for this kind of computation model have been developed and published by Selby and McClatchy (10), (11). These programs allow computation of the spectral or broad band solar intensity for any point in the atmosphere or on the ground. Also the extinction between two points of the lightpath produced by the different sources of attenuation can be obtained. These results are available for 6

different atmospheric models. Furthermore, other model atmospheres may be fed into the programs.

B 1.2 Computation of direct solar intensity by means of integral values of the extinction coefficients, which relate to the whole wavelength region considered.

Many authors apply formulae to compute S_1 , in which the integration with respect to wavelength is not carried out explicitly. In this case, the extinction coefficients represent integral or effective values, which embrace the whole shortwave region considered. The formulae are as follows:

$$S_1 = \rho^2 S_0 e^{-km} \quad (7)$$

$$k = k_R + k_p + k_a \quad (8)$$

The total extinction coefficient k represents the sum of all attenuation coefficients. Relations similar to those stated in section 1.1 for the spectral coefficients hold for the corresponding integral quantities: The coefficient for particle extinction k_p contains a component for particle scattering and one for particle absorption. The absorption coefficient k_a includes the absorption of the atmospheric gases water vapour, ozone, carbon dioxide and oxygen.

$$k_p = k_{ps} + k_{pa} \quad (9)$$

$$k_a = k_w + k_o + k(\text{CO}_2) + k(\text{O}_2) \quad (10)$$

The scattering and absorption coefficients k_R , k_w , k_o , etc. relate to the assumed values of the parameters b , w , x , relevant for each coefficient. The effective values of these coefficients can be determined from formulae (1) to (4), which hold for the corresponding spectral values. The effective values for k_p can be derived from the assumed turbidity model or, neglecting particle absorption k_{pa} , from the turbidity parameters β and α according to the Ångström/Schüepp relation (6).

As an example of this method we may consider Schulze's (12) approach to compute direct solar intensity S_1 (and sky- and global radiation) in different climates. Schulze's formula for S_1 is:

$$S_1 = \rho^2 S_0 q_a^m q_R^m q_p^m = S_0 q_R^{m \cdot T} \quad (11)$$

In this equation transmission factors such as $q_R = \exp(-k_R)$, etc. are applied in place of the corresponding extinction coefficients k_R , etc. Furthermore, the turbidity factor T according to Linke (13) is introduced. The author gives tables presenting the values for the transmission factors for different values of solar altitude h , precipitable water w and turbidity T . The influence of ozone and oxygen absorption is also considered in these figures.

In addition the resulting values of intensity of solar radiation (and sky- and global radiation) are presented in tables for different climates.

By means of the Linke turbidity factor T the total integral extinction coefficient k is expressed as a multiple of the integral Rayleigh scattering coefficient k_R .

A large number of data on the seasonal and geographical variation of the turbidity factor T is available. This factor characterizes in a most simple manner the total integral extinction coefficient, and is therefore frequently used as a principal parameter for computing approximate values of direct solar intensity S_1 . The following equation applies in this case:

$$S_1 = \rho^2 S_0 e^{-mk_R T} \quad (12)$$

For $\rho = 1$, and introducing the numerical values for S_0 and k_R according to Liebelt (14), this becomes:

$$S_1 = 1265 e^{-0.088 m \cdot T} \text{ Wm}^{-2} \quad (13)$$

Hocevar & Rakovec (15) proceed in principle according to equation (11). These authors apply two integral transmission coefficients q_a and q_s which take account of all absorption and scattering effects respectively. They assume fixed values $q_a = q_s = 0.90$, which are based on their own measurements. The values are found to be adequate for the scope of their calculations. Hocevar's and Rakovec's equation is adapted to yield daily sums of direct solar radiation on an inclined surface.

Revfeim (16) assumes one single total and integral transmission factor q according to the expression:

$$S_1 = S_0 q^m \quad (14)$$

The author presents a formula for the air-mass $m(z)$ which guarantees a higher degree of accuracy than the approximation $m(z) = \sec z$. The zenith angle z of the sun is determined by the astronomical parameters latitude, declination of the sun and hour angle. The transmission $q^{m(z)}$ which apart from S_0 determines the direct solar intensity S_1 is developed into a Fourier series with respect to the hour angle as variable. Furthermore, formulae for the computation of direct solar radiation on a surface of arbitrary orientation are presented.

B 1.3 Numerical approximation formulae to compute direct solar intensity S_1

Dogniaux's computing program (17)

Within the scope of a computer program to calculate radiation data for solar energy application Dogniaux presents the following formulae for direct solar intensity S_1 [G] on an arbitrarily oriented plane:

$$S_1 [G] = S_0 (n) e^{-k(h) m T(h, w, \beta) \cos \theta} \quad (15)$$

where

- n Number of the day $n = 1$ to 366
- $S_0(n)$ Extraterrestrial solar intensity for day No. n , corrected for the variable earth-sun distance, represented as a 3-term Fourier series with respect to the variable $n \frac{2\pi}{366}$
- $k(h)$ Total integral extinction coefficient, given by a 3-term Fourier series with the variable h
- $m(h)$ Airmass, given by a formula in dependence on solar altitude h , air pressure b or altitude a.s.l. respectively

$T(h, w, \beta)$ Turbidity factor by Linke, given by a formula depending on solar altitude h , precipitable water w and on the turbidity factor β according to Ångström/Schüepp

θ Angle of incidence of direct solar radiation on the receiver plane.

Dogniaux's computing program includes furthermore sky and global radiation and provides formulae for all auxiliary quantities needed.

Perrin de Brichambaut (18) in his work on the estimation of radiation data gives, among others, a formula for the mean daily values S_1 [H,d,10d] of solar radiation on a horizontal plane, averaged over a period of 10 days (no clouds):

$$S_1 [H,d,10d] = G_0 [H,d,10d] \exp \left[\frac{-T}{7.5} \cos^{0.75} (\phi - \delta) \right] \quad (16)$$

where G_0 [H,d,10d] stands for the daily mean of extraterrestrial global radiation, whereby $G_0 [H,d,10d] = S_0 [H,d,10d]$. The author presents an expression for computing G_0 [H,d,10d] depending on the astronomical parameters ϕ , δ and the extraterrestrial solar intensity S_0 . Also a formula for determining T from the values of the parameters w , β , ϕ and δ is given.

B 1.4 Computation of direct solar intensity from measured or theoretically estimated values of global and sky radiation.

A rather large number of investigations on radiation carried out for practical purposes are mainly concerned with global radiation on an inclined surface. In many cases results of measurements of global radiation, and partly also of sky radiation, on a horizontal surface serve as input values. Some authors use theoretically estimated values of sky radiation together with measured data on global radiation. The intensity $S[N]$ of direct solar radiation, which must be known for determining the solar component on inclined planes, is obtained by the relation:

$$S[N] = (G[H] - H[H]) / \sin h \quad (17)$$

This procedure is most frequently applied for mean values of radiation averaged over an hour or longer periods with arbitrary cloud conditions prevailing. This case is considered below in section 2.

If measured values $G_{1M}[H]$ of global radiation on a horizontal plane are available for cloudless sky, and if sky radiation $H_1[H]$ is theoretically estimated by an expression containing solar intensity $S_1[N]$ as variable, we may write:

$$G_{1M} = S_1 \sin h + H(S_1) \quad (18)$$

Solving this equation for S_1 the direct solar intensity can be derived. A procedure of this kind is applied by Basnet (19) in his report on the estimation of solar radiation falling on vertical surfaces from measurements on a horizontal surface.

B 2. COMPUTATION OF DIRECT SOLAR INTENSITY S FOR ARBITRARY CLOUD CONDITIONS

B 2.1 Formulae involving the relative sunshine duration σ as parameter.

Many authors apply the following relation to take account of the influence of clouds on direct solar intensity:

$$S_2 [N, \Delta t] = S_1 [N, \Delta t] \sigma [\Delta t] \quad (19)$$

A sufficiently long time interval Δt must be chosen to give the parameter $\sigma(\Delta t)$ a reasonable statistical significance. As a rule $\sigma(\Delta t)$ relates to an hour or a day and represents a mean over periods of a decade or a month.

Robinson & Schüepp (5) use relation (19) with hourly values of direct solar intensity and relative sunshine duration. Hocevar & Rakovec (15) apply the same procedure.

Relation (19) is based on the assumption that solar radiation is either temporarily shaded off or not influenced by the clouds. However, the effect of the clouds can assume any degree in between these extremes. Also,

the method actually applied for measuring sunshine duration should be considered. An improved approximation can be expected by replacing the parameter σ in (19) by an empirical function $f(\sigma)$, derived from statistical data on related values of S_2 and σ .

Dogniaux (17) gives a formula for hourly values $S(H, h, lp)$ of solar radiation on a horizontal surface averaged over a long period:

$$S_2 [H, d, lp] = S_1 [H, h, lp] \sigma^n [h, lp] \quad (20)$$

The exponent n is derived from local data for the place of observation in question. It is assumed here that the values of relative sunshine duration are randomly distributed over all hours, if a sufficiently long reference period, say 15 years, is considered. The daily values $\sigma[d, lp]$ can also be used in the same way as hourly values $\sigma[h, lp]$, so that we may put $\sigma[h, lp] = \sigma[d, lp]$ in relation (20).

Perrin de Brichambaut (18) considers daily means of direct solar radiation on a horizontal surface averaged over a decade and gives two alternative relations as follows:

$$S_2 [H, d, 10d] = S_1 [H, d, 10d] \frac{3\sigma}{5 - 2\sigma} \quad (21)$$

$$S_2 [H, d, 10d] = S_1 [H, d, 10d] \frac{\sigma}{5} (2\sigma + 3) \quad (22)$$

here σ represents the decadic mean $\sigma[d, 10d]$ of relative sunshine duration. These relations are based on the assumption, that the value of the turbidity factor T is the same for cloudless and partly clouded days, and that the value of σ is homogeneously distributed during the day. The author considers the latter assumption is true for reference periods > 10 days.

B 2.2 Further expressions for computing S_2

Relation (19) can be expressed in the form

$$S_2 = W \cdot S_1 = \rho^2 W \cdot S_0 e^{-km} \quad (23)$$

where the parameter W takes the place of σ . The total integral extinction coefficient k relates to the attenuation of direct solar

intensity by the cloudless atmosphere. Parameter W is determined from a large number of representative values S_{2M} measured in all kinds of cloud conditions.

This method is applied in principle by De Vos & De Mey (20), who assume a value of W independent of time of day. These authors interpolate the yearly variation of the parameter W according to the formula

$$W(n) = W_1 + W_2 \cos \frac{2\pi}{365} \cdot n = 0.345 - 0.136 \cdot \cos \frac{2\pi}{365} \cdot n \quad (24)$$

where n designates the number of the day.

The authors obtained S_0 and k from curves proposed by Dogniaux (21), which relate S_1 to airmass. Within the range of the relevant solar zenith angles Dogniaux's results can be approximated by straight lines corresponding to the relation: $\log S_1 = \log S_0'' - k'' \sec z$. The values of S_0'' and k'' thus obtained have been applied by De Vos & De Mey for their computations which include furthermore solar and diffuse radiation on inclined surfaces (see section 3.2, 6.2 and 8.1). Page (22) assumes in principle the following relation for mean hourly values of solar intensity $S_2[H,h]$ on a horizontal surface relating to arbitrary cloud conditions:

$$S_2[H,h] = \sigma[h] S_1^*[N,h] \exp(-k'm) \cdot \sin h \quad (25)$$

where $S_1^*[H,h]$ designates the direct solar intensity for the cloudless sky attenuated by Rayleigh scattering and atmospheric absorption, but not by aerosol absorption:

$$S_1^*[N,h] = \rho^2 S_0 \exp - (k_R + k_a) m \quad (26)$$

The value of m corresponds, say, to the middle of the hour considered. The coefficient k' represents an empirical parameter called "pseudoturbidity".

To explain the significance of relation (25) we may consider mean hourly values $S_2[H,h,p]$ of direct solar intensity for arbitrary cloudiness, averaged over a reference period p . The intensity prevailing during the intervals of time, for which the sun is not covered by clouds, is given by

$$S_1' = S_2[H,h,p]/\sigma[h] \quad (27)$$

If the clouds would either shade off the solar disc completely or leave it uncovered, the intensity S_1' could be expressed as

$$S_1' = S_1^*[N,h] \exp(-k_p \cdot m) \quad (28)$$

However, the assumption just mentioned is not realistic and k_p must be replaced by the empirical parameter k' which is adequately interpreted as pseudoturbidity. This consideration leads to the above expression (25) which in accordance with Page's procedure is formulated here for direct solar intensity $S_2[H,h]$ on a horizontal surface.

The pseudoturbidity k' can be computed from measured data of related hourly values of solar intensity $S_{2M}[H,h]$ and relative sunshine duration $\sigma[h]$. The formula reads

$$k'[h] = -\frac{1}{m} \log \frac{1}{\sigma[h] S_1^*[N,h] \sin h} S_{2M}[H,h] \quad (29)$$

According to this procedure Page derives the mean daily variation of the pseudoturbidity for each month. In place of the hourly value $\sigma[h]$ the author assumes a fixed mean value $\bar{\sigma}$ which is determined for the month in question. Based on these figures the mean daily variation of $k'[h]$ is obtained from formula (29) for each month. Proceeding from the values of $\bar{\sigma}$ and $k'[h]$ Page computes the monthly means of the hourly intensity $S_2[H,h,m]$ according to (25), as well as the daily sum $S_2[H,d,m]$ for Kew.

B 3. COMPUTATION OF SKY RADIATION ON A HORIZONTAL SURFACE FOR THE UNCLOUDED SKY

B 3.1 Computation of $H_1[H]$ based on a rigorous solution of the problem of multiple scattering in the atmosphere

A most general treatment of multiple scattering, its mathematical problems and methods of solution has been given in 1950 by Chandrasekhar (23) in his fundamental work "Radiative Transfer". A large number of computations of sky radiation from the unclouded sky have since been based on Chandrasekhar's theory:

In 1952 Sekera et al. (24) published the results of a computer program to calculate spectral and integral intensity and polarisation (Stokes' parameters) for solar, sky and global radiation. These results relate to an idealized model atmosphere assumed free of absorbing components and aerosols (Rayleigh atmosphere). The values of intensity are presented in dependence on different values of $\cos z$ and Rayleigh optical thickness, which combines the parameters of wavelength and altitude a.s.l. In a further program, Sekera et. al. (25) computed the values of intensity and all Stokes' parameters for sky radiance along different meridians of the hemisphere. Later on, the theoretical computations were extended step by step by Sekera and other authors to more realistic model atmospheres taking account of ozone absorption and aerosol scattering. Some of this work is devoted to the spectral intensity of sky radiation in the near ultraviolet: Dave & Furukawa (26), Braslau & Dave (27), Green et al. (28).

One recent computing program for solar, sky and global intensity, carried out by Dave & Braslau (29a, 29b, 30, 31) is judged by these authors as the most thorough approach undertaken up to now. The results are given for a number of different model atmospheres and turbidity models and their combinations. In one case a cloud layer is included.

Nagel, Quenzel et al. (74) published the results of extensive theoretical computations presented in a comprehensive set of tables titled: "Daylight Illumination-Color-Contrast Tables for Full-Form Objects". This work includes tables on the distribution of spectral sky radiance in $\text{Wm}^{-2}\text{ster}^{-1}\mu^{-1}$ over the hemisphere ($\Delta\zeta=5^\circ$, $\Delta\alpha=9^\circ$) for 10 wavelengths from 400 nm to 720 nm. In addition, the resulting values of global irradiance, expressed in $\text{Wm}^{-2}\mu^{-1}$ are given. In selecting the content of the tables to be presented the authors put the main emphasis on photometric quantities, such as luminance distribution over the sky (2160 reference points) and the quantities named in the title.

These theoretical results derived from a (practically) rigorous solution of the prob-

lem of multiple scattering in the atmosphere are obtained by means of elaborate and costly computer programs. Simpler procedures to determine approximate values of sky intensity $H_1[H]$ will be discussed in the following section.

B 3.2 Simpler methods for computing approximate values of sky intensity $H_1[H]$ derived from considerations based on atmospheric physics

Berlage's formula (32). As a first step Berlage (32) derives a formula for sky radiation H_1' of a Rayleigh atmosphere, assuming single scattering. Next, approximate expressions for sky radiation H_1 and global radiation G_1 are derived under the assumption of multiple scattering in a Rayleigh atmosphere. The author follows a procedure applied by Mecke (33), in which only the vertical component of the fluxes of scattered radiation are considered. Comparing the values of H_1 and H_1' , Berlage estimates the contribution $(1 - H_1'/H_1)$ of the multiple scattered radiation to the whole amount of scattered radiation. The author finds an increase of the contribution $(1 - H_1'/H_1)$ with decreasing values of the Rayleigh transmission coefficient q_R and solar altitude h . The contribution amounts to 12% for a transmission coefficient of $q_R = 0.4$ and a solar altitude of $h = 30^\circ$.

Berlages formula for sky intensity, relating to multiple scattering in a Rayleigh atmosphere reads:

$$H_1 = \frac{0.5 S_0 \sin h (1 - q_R^{\sec z})}{1 - 1.4 \log q_R} \quad (30)$$

The correction factor ρ^2 usually applied to S_0 is neglected in this case. Considering the influence of turbidity and absorption by means of Linke's turbidity factor T Berlage assumes the following relation for the total transmission factor q :

$$q = q_R^T \sec z \quad (31)$$

which leads to

$$H = \frac{0.5 S_0 \sin h (1 - q_R^T \sec z)}{1 - 1.4 T \log q_R} \quad (32)$$

This procedure takes account of the attenuation of direct solar radiation and scattered radiation by aerosol scattering and absorption. However, formula (32) is still based on the Rayleigh scattering function. Introducing the solar intensities S and S_0 we have

$$S_1 = S_0 q_R^T \sec z \quad (33)$$

or

$$T \log q_R = \sin h \cdot \log \left(\frac{S_1}{S_0} \right) \quad (34)$$

With (34) the final expression for H_1 becomes:

$$H_1 = \frac{0.5 \sin h (S_0 - S_1)}{1 - 1.4 \sin h \log \left(\frac{S_1}{S_0} \right)} \quad (35)$$

Expression (35) has been applied, among

others, by Basnet (19). Using measured hourly values $G_{1M} [H, h, m]$ as input data, Basnet considers the equation:

$$G_{1M} [H, h, m] = S_1 [N, h, m] \sin h + H_1 [H, h, m] \quad (36)$$

where $H_1 [H, h, m]$ is determined by formula (35). From this equation the intensity $S_1 [N, h, m]$ can be obtained first. Next, introducing S_1 into formula (35) the values of sky intensity $H_1 [H, h, m]$ are computed.

The intensity of sky radiation $H_1 [H]$ can be considered as a fraction $q(h)$ of the radiation scattered along the light path of direct solar radiation. The following relation has been proposed by Albrecht (34) and further supported by the work of Deirmendjian & Sekera and Sekera & Ashburn (36).

$$H_1 [H] = q(h) (S'_0 - \Delta S'_a - S) \sin h = q(h) (S'_a - S) \sin h \quad (37)$$

where

$$S'_0 = \rho^2 S_0 \quad (38)$$

$$S'_a = S'_0 - \Delta S'_a = S'_0 e^{-(k_a + k_{pa}) m} \quad (39)$$

Here S'_0 designates the extraterrestrial solar intensity, corrected for the variable sun-

earth distance. The quantity $\Delta S'_a$ represents the amount of solar intensity attenuated by the different components of atmospheric absorption, and S'_a stands for the solar intensity, reduced by absorption, but not by Rayleigh and aerosol scattering.

The parameter $q(h)$ can be determined from measured data $H_{1M} [H]$, obtained from very different values of turbidity and albedo. On the other hand, $q(h)$ may be derived from the theory of multiple scattering in the atmosphere. Schüepp derived from his measurements in Congo the relation

$$q(h) = 0.5 \sin^{1/3} h \quad (40)$$

The values of $q(h)$ vary between 0.5 and 0.67 in this region. A discussion of the parameter $q(h)$ and its variations is given by Robinson & Schüepp (5).

In a theoretical investigation Albrecht (34) examined global radiation in dependence on wavelength and solar altitude to explain the trend of measured data. The author derived a formula for computing global intensity and determined the proportion of upwards and downwards scattered radiation. Albrecht applied, among others, the above relation (37) as basic assumption.

Relation (37) is used e.g. by Robinson & Schüepp (5) and by Hocevar & Rakovec (15) to compute sky intensity. Robinson & Schüepp (5) follow the method discussed in section 1.1 to compute the values of the intensities S and S'_a . The numerical data for the evaluation of the intensity S'_a are presented in a table.

Hocevar & Rakovec (15) apply integral transmissions factors q_a and q_s for absorption and scattering respectively to express the difference $(S'_a - S)$ in relation (37). The factor q_s includes Rayleigh and aerosol scattering. Relation (37) reads in this case

$$H_1 [H] = q(h) \rho^2 S_0 [q_a^m - (q_a q_s)^m] \sin h \quad (41)$$

For lack of any better value the empirical parameter $q(h)$ is calculated according to relation (40) determined in Congo. Fixed values of $k_a = k_s = 0.9$ are considered adequate by the authors, as mentioned above in section 1.2.

Liebelt (14) also gives a formula for sky intensity H_1 [H] based on relation (37) and on the values of measured data:

$$H_1 \text{ (Wm}^{-2}\text{)} = 0.586.1265 (e^{-0.080} - e^{-0.088 m T}) \cdot \sin h \quad (43)$$

where $S_0 = 1265 \text{ Wm}^{-2}$

$$k_R = 0.088$$

$$k_a = 0.080$$

$$q(h) = \text{const.} = 0.586$$

The author used equation (43) to obtain his results on the distribution of sky radiance over the hemisphere.

A relation similar to (37) is used by De Vos & De Mey (20) to compute mean values of sky radiation averaged over longer periods, as will be discussed in section 6.4.

In an idealized atmosphere without aerosols, in which direct solar radiation is diffused according to the symmetric Rayleigh scattering function, the fractions of diffuse flux, scattered upwards and downwards respectively are equal. The intensity of sky radiation H_1 [H] amounts in this case to half of the intensity lost by direct solar radiation by scattering. Taking account of atmospheric absorption, the following expression for sky intensity may be set up under these assumptions:

$$H_1 [H] = 0.5 (S'_a - S) \sin h \quad (44)$$

This expression corresponds to relation (37) with a fixed value of $q(h) = 0.5$ for the empirical parameter $q(h)$.

Schulze (12) in applying expression (44), does not however assume an atmosphere free of aerosols. The difference $(S'_a - S)$ is obtained from the values of the transmission factors q_a , q_{pa} and q_R , as well as of the turbidity factor T given in tables by the author. Furthermore, Schulze assumes a value of particle absorption coefficient given by the relation $(q_{pa}^m) = 0.2(q_p^m)$, which means that aerosol absorption amounts to 20% of the total aerosol extinction. Expression (44) reads in this case:

$$H_1 [H] = 0.5 \rho^2 S_0 [q_a^m q_{pa}^m - q_R^{mT}] \sin h = 0.5 \rho^2 S_0 [0.2 q_p^m q_a^m - q_R^{mT}] \sin h \quad (45)$$

The values of sky intensity computed by means of this formula are presented in a table together with the results obtained for direct solar radiation and global radiation. These figures are given for different climates and solar altitudes.

In his work on radiation data Perrin de Brichambaut (18) gives a numerical approximation formula for the daily mean of sky intensity H_1 [H,d,10d] averaged over 10 days:

$$H_1 [H,d,10d] = G_0 [H,d,10d] \left[\frac{T}{31.6} \cdot \cos^{-0.5(\phi - \delta)} \right] \quad (46)$$

where $G_0 [H,d,10d] = S_0 [H,d,10d]$ represents the daily mean of extraterrestrial global radiation. As has been mentioned in section 1.3, the author gives a formula for computing $G_0 [H,d,10d]$ in dependence on the astronomical parameters ϕ and δ . In addition, an expression for determining the turbidity factor T from the values of w , β , ϕ and δ is presented.

The intensities S_1 , H_1 and $G_1 = S_1 \sin h + H_1$, which are determined by the formulae (16) and (46) respectively, are used by the author, among others, as basic parameters for computing daily means of solar, sky and global radiation for the arbitrarily clouded sky.

Valko's (65) formula for diffuse sky radiation H_2 [H] includes also the case of a clouded sky and is discussed further in section 6.5.

B 4. INTENSITY G_1 [H] OF GLOBAL RADIATION FROM THE UNCLOUDED SKY ON A HORIZONTAL SURFACE

For the unclouded sky global radiation is computed from its solar and sky component:

$$G_1 [H] = S_1 [N] \sin h + H_1 [H] \quad (47)$$

Mean values of global radiation related to the arbitrarily clouded sky may be derived directly from statistical data and expressed by means of numerical approximation formulae, as will be discussed in the next section.

B 5.COMPUTING METHODS FOR GLOBAL RADIATION ON A HORIZONTAL SURFACE WITH AN ARBITRARILY CLOUDED SKY

Mean values of global intensity G_2 [H] for average cloud conditions serve in many cases as parameter in expressions for estimating the corresponding values of sky intensity H_2 [H]. Therefore, global intensity is dis-

cussed first in this section. Sky intensity for the clouded sky is considered next in section 6.

B 5.1 Methods in which relative sunshine duration σ , amount of cloud c , or characteristic ratios Q are used as parameters

Table 1
Relations for computing G_2 [H]. The parameters and functions I, Q, f(σ) applied by different authors.

Author	I	f(σ)	f(c)	Q	Formula
Perrin de Bri- chambaut (18)	G_1 [H]	$\sqrt{2\sigma+1} - 0.72$ $a+(1-a)\frac{\sqrt{\sigma(\sigma+1)}}{2}$ for France: a=0,2 $\frac{1}{5}[1 + \sqrt{2\sigma(\sigma+1)}]$ $\sigma = \sigma [d,10d]$			$G_2[H,d,m] = G_1[H,d,m]$ • f(σ) (49)
Tricaud (37)	G_{1max} [H]	$\sigma K(K-1)$ K dependent on site and season $\sigma = \sigma [d,m]$			$G_2[H,d,m] = G_{1max}[H,d,m]$ • f(σ) (50)
Page (38)	G_0 [H]	$a + b\sigma$ $\sigma = \sigma [d,m]$			$G_2[H,d,m] = G_0[H,d,m]$ • f(σ) (51)
Dogniaux (17)	G_1 [H]	$a + b\sigma$ $\sigma_1 = \sigma [d,10d]$ $\sigma_2 = \sigma [d,1p]$			$G_2[H,d,10d] =$ $G_1[H,d,10d] f(\sigma_1)$ (52a) $G_2[H,d,1p] =$ $G_1[H,h] f(\sigma_2)$ (52b)
Liu & Jordan (39)	G_0 [H]			$\frac{G_2[H,d,m]}{G_0[H,d,m]}$	$G_2[H,d,m] = G_0[H,d,m] Q$ (53)
Schulze (12)	S_0 [H]		$1 - 0.55c$ $- 0.25c^4$	$Q_0 f(c)$	$G_2[H,m] = S_0[H,m]$ • (1-0.55c - 0.25c ⁴)Q ₀ (54a)
			$1 - 0.5c$ $- 0.3c^{10}$	$Q_0 = \frac{G_1[H]}{S_0[H]}$	$G_2[H,d] = S_0[H,d]$ • (1-0.5c - 0.3c ¹⁰)Q ₀ (54b)

Many authors derive empirical relations between mean values of global radiation $G_2[H]$ and relative sunshine duration σ . These relations include furthermore a radiation component I which may represent, say, the intensities $G_0[H]$, $G_1[H]$, etc. Also, characteristic ratios Q are applied and the amount of cloud c may replace the parameter σ . In general:

$$G_2[H] = I f(\sigma) \text{ or} \quad (48a)$$

$$G_2[H] = I \cdot Q \quad \text{or} \quad (48b)$$

$$G_2[H] = I f(c) Q \quad (48c)$$

The parameters and quantities applied, as well as the relations derived by different authors are shown in Table 1.

The following remarks concerning the formulas in Table 1 may be added:

Perrin de Brichambaut

The author assumes, that the duration of sunshine is homogeneous during the day, which is true for periods ≥ 10 days. The mean intensity $G_1[H, d, 10d]$ is determined by formulas (16) (46) and (47).

Tricaud

The author gives values for the constant K , obtained for different regions in France.

Page

The author gives tables of monthly mean values of $G_2[H, d, m]$ at intervals of 10° from $60^\circ N$ to $40^\circ S$ and values of the constants a and b for various sites in England and other countries.

Dogniaux

The local parameters a and b are deduced from analysis of data of global radiation and sunshine duration for the place of observation. Formula (52a) is valid for a reference period not shorter than 10 days. The author assumes that for a long reference period lp , say 15 years, sunshine duration can be statistically considered as randomly distributed over the hours of the day. It may be permissible then in most cases to apply to the irradiation received during a given hour of the day the long period daily values $\sigma_2 = \sigma[-d, lp]$ as the reduction factor, however orien-

tated or inclined the receiving surfaces may be. Formula (52b) is based on part of these assumptions.

Schulze

These formulas are valid for Hamburg and relate to monthly mean values (54a) and single day values (54b) of global radiation respectively.

B 5.2 Further formulas for computing global radiation $G_2[H]$ on a horizontal surface for average cloud conditions

The relations considered in this section have been derived by different authors and are quoted and critically examined in a paper published by Goldberg, Klein & McCartney (40). These relations are numerical approximation formulae representing statistical data.

The following three formulae are expressed cept for the designations G_2 and S_0 , which correspond to the symbols applied in the present paper.

Goldberg & Klein (41)

$$G_2[H, d] = S_0[H, d] [(1 + e^{-m^*R}) \cdot e^{-m^*(\alpha_{O_3} + \tau) + 0.1}] F_c \quad (55)$$

S_0 Daily total extraterrestrial radiation on a horizontal surface

R Integral Rayleigh extinction coefficient, $R = 0.104$

α_{O_3} Integral ozone absorption coefficient
 $\alpha_{O_3} = 0.045$

x Amount of ozone. A mean value of $x = 0.30$ cm is assumed

m^* Effective air mass for the day, as a function of the minimum air mass m , where $m = \sec z$ at noon.
 $m^* = 0.346 + 1.011 m + 0.0786 m^2$

τ Opacity plus albedo effects

F_c Correction for average cloud cover

Reddy (42)

$$G_2[H, d, m] = K(1 + 0.8 S)(1 - 0.2t) h^{-\frac{1}{2}} \quad (56)$$

G_2 Average insolation on a horizontal surface in $\text{cal cm}^{-2} \text{ day}^{-1}$
 $K = (\lambda N + \bar{v}_{ij} \cos \phi) \cdot 10^2$
 ϕ Latitude in deg.
 $\lambda = 0.2 / (1 + 0.1\phi)$
 N Mean daylength for the month in hours
 \bar{v}_{ij} Seasonal factor
 n Mean hours of bright sunshine per day for the month
 $S = n/N$
 t Proportion of days with rain for the month
 h Mean relative humidity for the month

Barbaro et al. (43)

$$G_2[H,m] = K S_m^{1.24} h_n^{-0.19} + 10550(\sin h_n)^{2.1} + 300(\sin h_n)^3 \quad (57)$$

$G_2[H,m]$ Monthly global radiation in cal cm^{-2}
 S_m Monthly sunshine duration in hours
 h_n Noon altitude of the sun on the 15th of the month
 K Zone factor (zone 1 = 8, zone 2 = 9.5, zone 3 = 11).

Goldberg et al. (40) check the results obtained by means of the different relations. The authors compare the values of global intensity G_2 obtained for Miami, $25^{\circ}47'N$ latitude as well as for $30^{\circ}N$, $40^{\circ}N$ and $50^{\circ}N$ from the formulae (55), (56) and (57). Also, the agreement of these results with measured intensity is tested for Miami and $40^{\circ}N$. Included in this check is furthermore Liu & Jordan's formula (53) given in Table 1. As Goldberg et al. find, the results computed from the relations by Liu & Jordan, Goldberg

& Klein and Barbaro et al. agree fairly well with each other and with the measured reference data. On the other hand, the figures derived from Reddy's model do not agree very well with any of the other three models, except at $30^{\circ}N$ latitude. This is due to an insufficient amount of solar radiation measurements used for constructing the model (40). According to Goldberg et al. these comparisons show, "that average daily values of yearly insolation can be determined reasonably well by the use of simple models".

B 6. COMPUTING METHODS FOR SKY RADIATION ON A HORIZONTAL SURFACE FOR ARBITRARY CLOUD CONDITIONS.

B 6.1 Methods in which relative sunshine duration σ and characteristic ratios Q are used, among others, as parameters.

Relations similar to those considered in section 5.1 are applied to relate mean values of sky intensity $H_2[H]$ to relative sunshine duration and to other parameters. Expressions of the following general form are used by different authors:

$$H_2[H] = I f(\sigma) \quad (58a)$$

$$H_2[H] = I f(Q) \quad (58b)$$

$$H_2[H] = I Q \quad (58c)$$

$$H_2[H] = I Q f(\sigma) \quad (58d)$$

The functions and parameters used by different authors are listed in Table 2.

Remarks concerning the quantities and formulae in Table 2:

Page (38)

The author gives monthly means for $H_2[H,d,m]$ at intervals of 10° from $60^\circ N$ to $40^\circ S$, as well as for the constants c and d for different sites in England and other countries.

Page (22)

The function $f(\sigma)$ can be derived from the formulae in Table 1 and 2 relating to Page (38), whereby $e = ac + a^2d$, $f = bc + 2abd$ and $g = b^2d$. Page finds for Kew: $e = 0.153$, $f = 0.289$, $g = 0.244$.

Perrin de Brichambaut (18)

Assumption: The duration of sunshine is homogeneous during the day, which is true for

Table 2.

Simple formulae for computing $H_2[H]$. The functions and parameters I , Q , $f(\sigma)$, $f(c)$ are as applied by different authors.

Author	I	$f(\sigma)$	Q	$f(Q)$	Formula
Page (38)	$G_2[H]$		$\frac{G_2[H,d,m]}{G_0[H,d,m]}$	$c+dQ$	$H_2[H,d,m]=G_2[H,d,m] f(Q)$ (59)
Page (22)	$G_0[H]$	$e + f\sigma + g\sigma^2$ $\sigma = \sigma[d,m]$ For e, f, g see remarks added to this table			$H_2[H,d,m]=G_0[H,d,m] f(\sigma)$ (60)
Perrin de Brichambaut (18)	$G_2[H]$	σ or better $\frac{3\sigma}{3 - 2\sigma(\sigma-1)}$ $\sigma = \sigma[d,10d]$	$\frac{D_1[H,d,10d]}{G_1[H,d,10d]}$		$H_2[H,d,m]=G_2[H,d,m] Q\sigma$ or better (61a) $H_2[H,d,m]=G_2[H,d,m]$ $\cdot Qf(\sigma)$ (61b)
Perrin de Brichambaut (18)	$G_1[H]$	$0 = \sigma: 1/4$ $0,4 < \sigma < 0,8: 1/3$ $\sigma > 0,9: 1/3 - 1/5$ $\sigma = \sigma[d,10d]$			$H_2[H,d,10d]=G_1[H,d,10d]$ $\cdot f(\sigma)$ (62)
Tricaud (37)	$G_2[H]$	$0,84 - 0,78\sigma$ $\sigma = \sigma[d,m]$			$H_2[H,d,m]=G_2[H,d,m] f(\sigma)$ (63)
Liu & Jordan (44), (45)	$G_2[H,d]$ $G_2[H,d,m]$		$\frac{G_2[H,d]}{G_0[H,d]}$ $\frac{G_2[h,d,m]}{G_0[H,d,m]}$	see (44) and (45) $1,390 -$ $- 4,027Q +$ $+ 5,531Q^2 -$ $- 3,108Q^3$ see (49), (44), (45)	$H_2[H,d]=G_2[H,d] f(Q)$ (64a) $H_2[H,d,m]=G_2[H,d,m] f(Q)$ (64b)

periods of ≥ 10 days. The mean intensity $G_1[H, d, 10d]$ is determined by formulae (16), (46) in subsections 1.3 and 3.2 respectively.

Tricaud (37)

The values of global intensity $G_2[H, d, m]$ are determined by formula (50) in Table 1.

Liu & Jordan

Formula (64a) is derived from statistical data on daily values of irradiations and the corresponding ratio Q , whereas equation (64b) relates to monthly means of the daily values of these quantities. The functions $f(Q)$ for these two cases are represented in diagrams Fig. 8 of ref. (44) and Figs. 7 and 14 of ref. (45). The formula for $f(Q)$ in (64b) is taken from Klein's paper (49).

B 6.2. Sky radiation from the unclouded and clouded part of the sky is computed separately.

Robinson & Schüepp (5) compute the sky intensity $H_2^u[H]$ from the unclouded part and the intensity $H_2^c[H]$ from the clouded part of the sky separately. Relative sunshine duration serves as a parameter in the relations assumed for two different contributions:

$$H_2^c[H, h] = H_1[H, h] \sigma \quad (65a)$$

$$H_2^u[H, h] = G_1[H, h] f(\sigma) \quad (65b)$$

The values of $H_1[H, h]$ and $S_1[N, h]$ are determined by the author from the formulae (37), section 3.2 and (4), section 1.1. Global intensity $G_1[H, h]$ is obtained from the results for $H_1[H, h]$ and $S_1[N, h]$.

Hocevar & Rakovec (15) also consider the radiation of the unclouded and clouded part of the sky separately. These authors proceed from the following relations:

$$H_2^c[H, h, m] = H_1[H, h, m] \sigma \quad (66a)$$

$$H_2^u[H, h, m] = H_1[H, h, m] (1 - \sigma) f(z, \text{clouds}) \quad (66b)$$

where the mean hourly values of sky radiation $H_1[H, h, m]$ of the whole unclouded sky are computed from the formula (41) section 3.2. The empirical function $f(z, \text{clouds})$ can be expressed as

$$f(z, \text{clouds}) = a + bz - cz^2 \quad (67)$$

Proceeding from the data of Tverskoy (46) the authors give the values of the factors a , b and c for 6 different conditions of cloudiness. Formulae (65a) and (65b) demonstrate Hocevar & Rakovec's approach in principle. Actually, the author's formulae relate to mean daily values $H_2[G, d]$ of sky radiation on inclined surfaces.

Page (47) considers three parts of the sky, namely:

- 1) the unclouded part except the region near the sun with its increased intensity,
- 2) the region near the sun, and
- 3) the clouded part.

This model also serves to compute sky intensity $H_2[G]$ on inclined surfaces.

B 6.3 A relation of the form: $H_2[H, h, m] = \frac{K_1 + K_2 \cdot h}{W_m^{-2}}$ (68a)

Page (22) finds for the monthly means of hourly values $H_{2Kew}[H, h, m]$ at Kew an approximation formula reading:

$$H_{2Kew}[H, h, m] = 2 + 4.804 h \quad W_m^{-2} \quad (68b)$$

This relationship is based on data for Kew, 1965-75, excluding 1973.

Following a procedure applied by Page mean hourly values $H_{2site}[H, h, m]$ of sky intensity for any other site can be deduced. The ratio of the mean daily values of sky intensity relating to the site and to Kew respectively serves as a correction factor:

$$H_{2site}[H, h, m] = H_{2Kew}[H, h, m] \cdot \frac{H_{2site}[H, d, m]}{H_{2Kew}[H, d, m]} W_m^{-2} \quad (69a)$$

$$H_{2site}[H, d, m] = (2 + 4.804 h) \cdot \frac{H_{2site}[H, d, m]}{H_{2Kew}[H, d, m]} W_m^{-2} \quad (69b)$$

where the values of $H_{2site}[H, d, m]$ and $H_{2Kew}[H, d, m]$ are computed from the formulae (59) and (60) respectively given in Table 2.

B 6.4 The relation assumed by De Vos & De Mey

An amount $S_2 \sin h$ of the downward solar flux per m^2 reaches the ground. The difference $\Delta S_2 = (S_0 - S_2) \sin h$ represents the amount which is attenuated by atmospheric scattering and absorption. Considering relation (23) for S_2 assumed by De Vos & De Mey (20) we have:

$$\Delta S_2 = (S_0 - S_2) \sin h = S_0 \sin h \cdot [1 - W \exp(-k_a m)] \quad (70)$$

A fraction U of the direct solar flux lost by atmospheric attenuation reaches the ground as sky radiation. The authors postulate the following relation for H_2 :

$$H_2[H] = U \Delta S_2 = U S_0 \sin h [1 - W \exp(-k_a m)] \quad (71)$$

As has been mentioned in section 2.2 above the empirical factor W takes account of the attenuation of direct solar radiation by Rayleigh scattering and extinction by aerosols and clouds. The values of W for each day of the year is determined by the interpolation formula (24). The factor U ranges between 0.2 and 0.6 according to Dave et al. (48). De Vos & De Mey assume a value of $U = 0.4$ for their computations.

B 6.5 Valko's formula for diffuse irradiance $H_2[H]$ on a horizontal surface.

Considering the data collected during a five year period at Locarno-Monti (Southern Switzerland) Valko (65) examined the relationship between diffuse sky radiation and solar altitude, turbidity and cloudiness. By grouping the data by months it was possible to show quantitatively the seasonal dependence of diffuse intensity on surface albedo and on direct reflection from the slopes. The author applied the so-called coaxial method for the graphical derivation of the relationship considered. This procedure leads to a display of the results in a multicurve-graph for practical use. The relationship is approached by the general expression (see Valko (63)):

$$D_2[H] = Y [a_1 (\sin h)^{b_1} + a_2 (\sin h)^{b_2} \cdot (\beta - d_1)^{b_3}] [a_3 10^{b_4 c_1} + a_4 c_2 + d_2]$$

where a_i, b_i, d_i are appropriate constants and c_1 and c_2 designate the cloudiness for low + middle and high clouds respectively. The seasonal factor Y assumes different values for spring, summer, autumn and winter. The terms in the first bracket present the contributions of Rayleigh scattering and aerosol scattering. The terms in the second bracket take account of the influence of cloudiness, which has been established up to $c_1 = 0.8$ with $c_2 = 0$ and up to $c_2 = 0.4$ with $c_1 = 0$. For Locarno-Monti the formula reads:

$$D_2[H] = Y [57(\sin h)^{0.35} + 3.7(\sin h)^{0.66} \cdot (10^3 \beta - 12)^{0.81}] [0.11 \cdot 10^{0.17 c_1} + 0.035 \cdot c_2 + 0.89]$$

with $Y = 1.04$ (spring), 0.89 (summer), 0.96 (autumn) and 1.11 (winter). The reliability of the statistical results obtained for Locarno-Monti and represented by this formula was checked against a series of measured values not involved in the statistics. For cloudless sky the results are compared with those gained by other authors.

B 7. IRRADIANCE OF DIRECT SOLAR RADIATION ON INCLINED SURFACES.

B 7.1 Normal component of direct solar radiation on inclined surfaces when the sun is not covered by clouds.

Proceeding from the values of direct solar intensity $S_1[N]$ solar irradiance on inclined surfaces can be computed by means of the formulae of spherical astronomy and other trigonometrical relations:

$$S_1[G] = S_1[N] \cos \theta \quad (72)$$

$$\cos \theta = f(h, a, s, \gamma) \quad (73)$$

where h, a and θ designate solar altitude, azimuth and angle of incidence with respect to the receiving surface. The inclination s and the azimuth γ determine the orientation of the surface. The angle γ is defined as the azimuth angle of the normal to the receiving surface.

Considering mean values of solar radiation relating to a relatively short period we have e.g. for hourly means:

$$S_1[G,h] = S_1[N,h] \overline{\cos \theta} \quad (72a)$$

$$\overline{\cos \theta} = f(\bar{h}, \bar{a}, s, \gamma) \quad (73a)$$

To take account of the variation of the parameters h and a , hourly means of effective values only must be applied for $\overline{\cos \theta}$ or the parameters h and a respectively.

The well known relations for computing solar altitude h and azimuth a from the astronomical parameters δ , ϕ and τ may be repeated here:

$$\sin h = \sin \delta \sin \phi + \cos \delta \cos \phi \cos \tau \quad (74)$$

$$\sin a = \frac{\cos \delta \sin \tau}{\cos h} \quad (75a)$$

$$\cos a = \frac{-\sin \delta \cos \phi + \cos \delta \sin \phi \cos \tau}{\cos h} \quad (75b)$$

The parameters a and τ , as well as γ are counted clockwise from the S-meridian from 0° to 360° . Solar altitude h is limited to the interval $\pm 90^\circ$. The azimuth a is not determined uniquely by $\sin a$ or $\cos a$. However, the proper value is evident from the geometrical significance of the parameters a and τ .

The angle of incidence θ of solar radiation with respect to the receiving surface is given by formula:

$$\cos \theta = \cos s \sin h + \sin s \cos h \cos (a - \gamma) \quad (76)$$

A negative sign of $\cos \theta$ indicates that the side of the surface corresponding to azimuth angle γ is not exposed to direct solar radiation.

B 7.2 Normal component of direct solar radiation on inclined surfaces with arbitrary conditions of cloudiness.

Various methods for computing mean values of direct solar intensity $S_2[N]$ or its verti-

cal component $S_2[H]$ for average cloudiness have been discussed in section 2. The normal component $S_2[G]$ with respect to an inclined surface is determined by the same formulae (72), (73) which apply for unclouded conditions. On account of the variation of the parameters h and a and of the clouds the averaging period should neither be too long nor too short. Hourly values of intensity are considered in most cases:

$$S_2[G,h] = S_2[N,h] \overline{\cos \theta} \quad (77a)$$

$$S_2[G,h] = S_2[H,h] \overline{\cos \theta} / \overline{\sin h} \quad (77b)$$

The relations discussed in section 2 may serve to compute the intensities $S_2[N,h]$ and $S_2[H,h]$, as far as they relate to hourly values. If hourly values of global and sky radiation are available from measurements the corresponding figures for $S_2[N,h]$ of solar intensity may be derived directly by means of the relationship

$$S_2[N,h] = (G_{2M}[H,h] - H_{2M}[H,h]) / \overline{\sin h} \quad (78)$$

For daily means of direct solar radiation on inclined surfaces we have in place of formula (77a):

$$S_2[G,d,p] = F_N(\phi, \delta, s, \gamma) S_2[N,d,p] \quad (79a)$$

Factor $F_N(\phi, \delta, s, \gamma)$ represents the weighted mean of $\overline{\cos \theta}$ extended over the day, using $S_2[N,h]$ as the weighting function. If this procedure is based on the vertical component $S_2[H,d,p]$ of solar radiation the formula becomes:

$$S_2[G,d,p] = F_H(\phi, \delta, s, \gamma) S_2[H,d,p] \quad (79b)$$

In this case the factor $F_H(\phi, \delta, s, \gamma)$ stands for the weighted mean of $\overline{\cos \theta} / \overline{\sin h}$.

Values of the factor F_H have been determined and plotted by Perrin de Brichambaut (18) for each month as a function of latitude for various inclined surfaces. This author applies formula (61a) and (61b) of Table 2, subsection 6.1 to compute the vertical component $S_2[H,d,10d]$.

Values of F_H have furthermore been computed by Page (38) for monthly daily solar irradiance. These results relate to 14 dates in the year and to 5 different orientations of the receiving surface. The figures are tabulated in 10° intervals from $60^\circ N$ to $40^\circ S$.

B 7.3 Liu & Jordan's procedure

Liu & Jordan's (44) computing methods allow one to determine global irradiance on South orientated planes from values of global irradiance on a horizontal surface. Conversion factors to be applied to the direct solar component and sky component on a horizontal surface are derived among others (see section 10). Only the procedure for the direct solar component is discussed here.

The authors consider the ratio

$$P_{S_0}[d] = S[G,d] / S[H,d] \quad (80)$$

between the daily values of the direct solar component on inclined and horizontal surfaces respectively. Only South facing surfaces are taken into account. Considering first the instantaneous values of solar irradiance above the atmosphere we have:

$$P_{S_0}[t] = S_0[G,t] / S_0[H,t] = \frac{\cos \theta}{\cos z} \quad (81)$$

$$\text{where } \cos z = \sin h = \sin \delta \sin \phi + \cos \delta \cos \phi \cos \tau \quad (74)$$

A surface located at latitude ϕ inclined at an angle s towards south is parallel to a horizontal surface located at latitude $(\phi-s)$. Therefore:

$$\cos \theta = \sin(\phi-s) \sin \delta + \cos(\phi-s) \cos \delta \cos \tau \quad (82)$$

for South facing surfaces

Daily values $P_{S_0}[d]$ of the ratio P_{S_0} are determined by integrating the instantaneous extraterrestrial irradiances over the period for which the sun is over the horizon. This involves an integration over all hour angles τ from $-\tau_s$ to τ_s where τ_s is the sunset hour angle:

$$\cos \tau_s = -\tan \phi \tan \delta \quad (83a)$$

for a horizontal surface

$$\cos \tau'_s = -\tan(\phi-s) \tan \delta \quad (83b)$$

for an inclined surface facing South

The expression for the ratio $P_{S_0}[d]$ becomes

$$P_{S_0}[d] = \frac{\int_{-\tau'_s}^{\tau_s} \cos \theta d \left(\frac{24\tau}{2\pi} \right)}{\int_{-\tau_s}^{\tau_s} \cos z d \left(\frac{24\tau}{2\pi} \right)} \quad \text{for } \tau'_s < \tau_s$$

During the winter half of the year ($\delta < 0$, $\tau_s < \frac{\pi}{2}$) the sunset hour angle τ'_s for the inclined surface is larger than τ_s . For this period both integrals in (84) are extended from $-\tau_s$ to $+\tau_s$.

Considering relations (74), (82), (83) and performing this integration the authors obtain:

$$P_{S_0}[d] = \frac{\cos(\phi-s)}{\cos \phi} \frac{\sin \tau'_s - \tau'_s \cos \tau'_s}{\sin \tau_s - \tau_s \cos \tau_s} \quad (85a)$$

for $\tau'_s < \tau_s$

$$P_{S_0}[d] = \frac{\cos(\phi-s)}{\cos \phi} \frac{\sin \tau_s - \tau_s \cos \tau_s}{\sin \tau'_s - \tau'_s \cos \tau'_s} \quad (85b)$$

for $\tau'_s > \tau_s$

Relation (83b) has been applied to express $P_{S_0}[d]$ according to the relations (85a), (85b) and to eliminate δ . The argument τ'_s in (85b) has nothing to do with the integration which is extended from $-\tau_s$ to τ_s in this case.

At the equinox ($\delta = 0$ and $\tau_s = \tau'_s = \frac{\pi}{2}$) these relations reduce to

$$P_{S_0}[d]_{\text{equinox}} = \frac{\cos(\phi-s)}{\cos \phi} \quad (86)$$

The direct solar irradiances $S[G,d]$ and $S[H,d]$ at ground level include the total integral transmission $q^m(\tau)$, which depends on the hour angle. The expression for the ratio $P_S[d]$ reads in this case:

$$P_S[d] = \frac{\int_{-\tau_s}^{\tau_{sr}} q^m(\tau) \cos \theta d\left(\frac{24\tau}{2\pi}\right)}{\int_{-\tau_s}^{\tau_{sr}} q^m(\tau) \cos z d\left(\frac{24\tau}{2\pi}\right)} \quad (87)$$

At the equinox ($\delta = 0$) the expressions for $\cos z$ and $\cos \theta$ (for South facing surfaces) become:

$$\cos z = \cos \phi \cos \tau \quad (88)$$

$$\cos \theta = \cos(\phi - s) \cos \tau \quad (89)$$

If these formulae are introduced into (87) the integrals in the numerator and denominator cancel and the equation for $P_S[d]$ is reduced to:

$$P_S[d]_{\text{equinox}} = \frac{\cos(\phi - s)}{\cos \phi} = P_{S_0}[d]_{\text{equinox}} \quad (90)$$

Thus, at the equinox the value of the ratio $P_S[d]$ is equal to the value of the extraterrestrial ratio $P_{S_0}[d]$ predicted by the formulas (85a) and (85b) setting $\delta = 0$. The formulae (85a) and (85b) give therefore the correct result, although they have been derived for extraterrestrial solar radiation only. Liu & Jordan recommend for this reason that the equations (85a) and (85b) be used to compute the ratio $P_S[d]$ as a first approximation for any date of the year. Consequently, taking $P_{S_0}[d]$ as conversion factor for the direct solar component we have:

$$S[G,d] = P_S[d] S[H,d] \quad (91)$$

$$P_S[d] = P_{S_0}[d] = \frac{\cos(\phi - s)}{\cos \phi} \quad (92)$$

As the transmission $q^m(\tau)$ in expression (87) applies equally to extinction by the turbid atmosphere and by cloud the author's approximation is valid also for the clouded sun, provided that some direct solar radiation penetrates the cloud layer at all. However, the authors emphasize that except during the time of the equinoxes, the application of these equations should be made on a long term statistical average basis only. It should also be expected that the use of this approximate procedure will give best results when the solar declination is not large and probably becomes less reliable for times during the solstices.

B 7.4 Klein's extension of Liu & Jordan's method

Klein (49) extended Liu & Jordan's formula for South facing surfaces to surfaces of a wide range of orientations. This adaptation to the more general case relates to the direct solar component only. Proceeding from Liu & Jordan's approach the author considers the ratio $P_{S_0}[d] = S_0[G,d]/S_0[H,d]$ of the daily extraterrestrial solar irradiances taking account of an arbitrary orientation of the receiving surface westwards ($\gamma > 0$) or eastwards ($\gamma < 0$) of the southern meridian. An expression similar to (84) applies. The integrations are extended over all hour angles for which the sun is in front of the inclined surface and above the horizon. The rather involved equations derived for $P_{S_0}[d]$ will not be stated here in detail. Formally these equations may be set out as follows:

$$P_{S_0}[d] = f(s, \gamma, \phi, \delta, \tau_{sr}, \tau_{ss}, \tau_s) \quad (93)$$

$$\begin{matrix} \tau_{sr} \\ \tau_{ss} \end{matrix} = f(s, \gamma, \phi, \delta) \quad (94)$$

where the symbols τ_{sr} , τ_{ss} , $-\tau_s$, τ_s designate sunrise and sunset hour angles for the inclined and horizontal surface respectively. The equations for τ_{sr} and τ_{ss} are the same except for the sign of function f and of a term included in f . Formula (83a) applies for τ_s .

Following Liu & Jordan's procedure the ratio $P_{S_0}[d]$ thus computed is taken as an approximation to the actual ratio $P_S[d]$ relating to the irradiances existing below a real atmosphere. The limitations of this approximation, emphasized by Liu & Jordan, are pointed out above in subsection 7.3.

B 8. COMPUTATION OF DIFFUSE RADIATION ON INCLINED SURFACES

B 8.1 Assumption of an isotropic distribution of sky radiation and of the radiation reflected from the ground.

The diffuse radiation on an inclined surface includes the sky radiation $H[G]$ and a

contribution $R[G]$ of the radiation reflected from the ground. Both components depend on the amount of sky and surrounding ground, which can be seen from the surface. Many authors apply a rather crude approximation assuming an isotropic distribution of sky radiance and of the radiance of the reflected radiation. The formula for sky intensity $H[G]$ on an inclined surface of arbitrary orientations reads in this case:

$$H[G] = H[H] \cos^2 \frac{s}{2} = \frac{1}{2} H[H] (1 + \cos s) \quad (95)$$

where s designates the angle of inclination. Assuming furthermore isotropy for the radiation reflected from the ground we have for $R[G]$

$$R[G] = R[H] \sin^2 \frac{s}{2} = \frac{1}{2} R[H] (1 - \cos s) \quad (96)$$

$$R[H] = A G[H] \quad (97)$$

The quantity $R[H]$ represents the total radiation reflected from the horizontal ground in dependence of albedo A and global irradiation $G[H]$. The sky and reflected components add up to the diffuse intensity $D[G]$:

$$D[G] = H[G] + R[G] = H[H] \cos^2 \frac{s}{2} + A G[H] \sin^2 \frac{s}{2} \quad (98)$$

The formulae (95) and (98) are equally applied for the unclouded sky and for situations with arbitrarily clouded sky.

Actually the distribution of sky radiance is not isotropic except in conditions with fog. Sky radiance of the unclouded sky increases towards the horizon and also assumes very high values near the sun in the case of higher turbidity. The distribution of reflected radiations is influenced by the different kinds of ground and orographic conditions. Clouds may enhance the anisotropy of sky radiation. The assumption of isotropy yields the best approximation for regularly distributed clouds or if average values of sky intensity for mean conditions of cloudiness are considered. The averaging period should be chosen sufficiently long, so that the distribution of clouds may be considered as homogeneous. The errors produced by the assumption of isotropic sky and reflected radiation have been investigated by Temps and Coulson, (50), (51) as will be discussed in subsection 8.6. The anisotropy of sky radi-

ation and its influence on the irradiation of inclined surfaces has further been studied by Page (38). Kondratyev (66) emphasizes the defects of the assumption of isotropy for all cases except the overcast sky.

Many authors base their computations or estimations of diffuse radiation $D(G)$ on the isotropy model, in spite of its obvious deficiencies e.g. Perrin de Brichambaut (18), Robinson & Schüepp (5), Hoyer & Rakovec (15), De Vos & De Mey (20) and others.

B 8.2 Computing sky radiation on inclined surfaces from the distribution of radiance over the hemisphere. General relationships.

Computing $H[G]$ from the distribution of sky radiance is, from a theoretical point of view, the most satisfactory method. The sky radiance $J(\zeta, \phi, h)$ depends on the zenith angle ζ and the azimuth ϕ of the element $\Delta \Omega$ of angular area considered, and is furthermore a function of other parameters such as solar altitude, turbidity and precipitable water.

The angle of incidence θ of a pencil of radiation from the direction of $\Delta \Omega$ with the normal to the receiving surface is given by the relation:

$$\cos \theta = \cos s \cos \zeta + \sin s \sin \zeta \cos(\phi - \gamma) \quad (99)$$

where s designates the inclination and γ stands for the azimuth of the surface. The contribution of the angular area $\Delta \Omega$ to sky intensity $H[G]$ amounts to

$$\Delta H[G] = J(\zeta, \phi, h) \cos \theta(\zeta, \phi) \sin \zeta \Delta \zeta \Delta \phi \quad (100)$$

For the total sky intensity $H[G]$ we have:

$$H[G] = \int_{\Omega'} J(\zeta, \phi, h) \cos \theta(\zeta, \phi) \sin \zeta d\zeta d\phi \quad (101)$$

The integration includes the whole angular area Ω' visible from the surface and determined by the condition $\cos \theta > 0$.

Often the relative radiance distribution $j(\zeta, \phi, h)$ with respect to zenith radiance $J_z(h)$ is considered in place of $J(\zeta, \phi, h)$

$$j(\zeta, \phi, h) = J(\zeta, \phi, h) / J_z(h) \quad (102)$$

The relations for sky radiation on inclined and horizontal surfaces are for this case respectively:

$$H[G] = J_z(h) \int_0^{\pi/2} \int_0^{2\pi} j(\zeta, \phi) \cos \theta \sin \zeta \, d\zeta \, d\phi \quad (103)$$

$$H[H] = J_z(h) \int_0^{\pi/2} \int_0^{2\pi} j(\zeta, \phi) \cos \zeta \sin \zeta \, d\zeta \, d\phi \quad (104)$$

If $j(\zeta, \phi, h)$ and $H[H]$ are known from measurements, zenith radiance $J_z(h)$ can be computed by means of relation (104).

Using the values J_z obtained by this procedure, sky radiation $H[G]$ on a surface of any orientation may be determined from relation (103).

Within the scope of daylight research the distribution of sky luminance \hat{J} and the relative luminance distribution \hat{j} with respect to zenith luminance \hat{J}_z is measured. If the radiometric quantities $H(\text{Wm}^{-2})$, $J(\text{Wm}^{-2}\text{ster}^{-1})$ and j are replaced by the corresponding photometric quantities $\hat{H}(\text{lx})$, $\hat{J}(\text{cd m}^{-2})$ and \hat{j} the relations (100) to (104) remain valid.

Radiometric quantities are converted into photometric quantities and vice versa by means of the luminous efficiency K . If I designates any of the radiometric quantities H, J, j etc. and I_λ stands for their spectral distribution the corresponding luminous efficiency is defined as

$$K_I (\text{lm W}^{-1}) = \frac{\hat{I}}{I} = \text{Km} \int_{380}^{780} V(\lambda) d\lambda / \int_0^{\infty} I_\lambda d\lambda \quad (105)$$

In this formula $K = 673 \text{ lmW}^{-1}$ represents the maximum of luminous efficiency related to the wavelength of maximum visual sensitivity and $V(\lambda)$ designates the normalized relative visual sensitivity function.

The following relations may be stated:

$$J_z(h) = \frac{\hat{J}_z}{K_z} \quad (106)$$

$$j(\zeta, \phi) = \hat{j}(\zeta, \phi) K'(\zeta, \phi) \quad (107)$$

$$K'(\zeta, \phi) = \frac{K_z}{K_j} \quad (108)$$

Introducing the photometric quantities \hat{J}_z , \hat{j} and K' into formulas (103) and (104) we have for $H[G]$ and $H[H]$:

$$H[G] = \frac{\hat{J}_z(h)}{K_z} \int_0^{\pi/2} \int_0^{2\pi} \hat{j}(\zeta, \phi) K'(\zeta, \phi) \cos \theta \sin \zeta \, d\zeta \, d\phi = J_z(h) \int_0^{\pi/2} \int_0^{2\pi} \hat{j} K' \cos \theta \sin \zeta \, d\zeta \, d\phi \quad (109)$$

$$H[H] = \frac{\hat{J}_z(h)}{K_z} \int_0^{\pi/2} \int_0^{2\pi} \hat{j}(\zeta, \phi) K' \cos \zeta \sin \zeta \, d\zeta \, d\phi = J_z(h) \int_0^{\pi/2} \int_0^{2\pi} \hat{j} K' \cos \zeta \sin \zeta \, d\zeta \, d\phi \quad (110)$$

It is assumed in many cases that the radiance distribution $J(\zeta, \phi)$ is proportional to the luminance distribution $\hat{J}(\zeta, \phi)$ which would mean $j(\zeta, \phi) = \hat{j}(\zeta, \phi)$ and $K' = 1$. In this approximation formula (103) read:

$$H[G] = J_z(h) \int_0^{\pi/2} \int_0^{2\pi} \hat{j}(\zeta, \phi) \cos \theta \sin \zeta \, d\zeta \, d\phi \quad (111)$$

An example is shortly discussed in the next section.

B 8.3 Dogniaux's computer procedure to calculate sky irradiance on inclined surfaces for cloudless and overcast sky.

Dogniaux (17) adopts the assumption $j(\zeta, \phi) = \hat{j}(\zeta, \phi)$ and applies in principle formula (111) to compute sky irradiation $H[G]$. Two cases involving different sets of functions for J_z and \hat{j} are considered by the author

Clear sky conditions:

Zenith radiance:

$$J_z(h, T) = (0.0023 h^2 + 0.0200 h + 1.3513) T \quad (112)$$

Sky brightness:

The relative sky luminance distribution by Kittler, normalized by CIE (52) and valid for small turbidity is applied.

$$\hat{j}(\zeta, \phi) = \frac{(C_1 + C_2 e^{-3\psi} + C_3 \cos^2 \psi) (1 - e^{-0.32 \sec \zeta})}{0.27385(C_1 + C_2 e^{-3z} + C_3 \cos^2 z)} \quad (113)$$

where ψ designates the angle between the angular area element $d\zeta d\phi$ and the sun.

$$\cos \psi = \cos \zeta \cos z + \sin \zeta \sin z \cos(\phi - a) \quad (114)$$

The values of the constants C_1, C_2, C_3 are given in the following table

	Kittler rel. small turbidity	Gusev rel. large turbidity
C_1	0,91	0,856
C_2	10	16
C_3	0,45	0,3

The values of the constants for a similar formula by Gusev (52) valid for relatively high turbidity are also included in the table.

Overcast sky

The Moon and Spencer distribution (53) is used by the author for this case:

$$J_z(h) = 18.5 \sin h \quad (115)$$

$$\hat{j}(h') = \frac{1}{3} (1 + 2 \sin h') \quad (116)$$

The integral in formula (111) is evaluated by numerical integration. The final formula reads:

$$H[G] = 0,0076154 \sum_{h_1=2,5}^{87,5} \sum_{\phi_1=2,5}^{357,5} J_1(h', \phi) \cdot \cos \theta_1 \cos h'_1 \quad (117)$$

where the angular elevation $h' = 90^\circ - \zeta$ is used in place of zenith angle ζ , and

$$J(h', \phi) = J_z(h, T) \hat{j}(h', \phi) \quad (118)$$

Dogniaux's computer procedure covers also direct solar intensity and global radiation for cloudless and clouded sky (see sections 1.3 and 2.1). Formulae for all astronomical and atmospheric parameters necessary for the computations are provided.

B 8.4 Liebelt's and Liebelt & Bodmann's results on luminance and radiance distribution of the unclouded sky and sky irradiance on a vertical surface facing South

In his comprehensive investigation Liebelt (14) carried out a large number of measurements on sky radiance distribution and their correlation. His work includes 330 records of

luminance and radiance, as well as of direct solar radiation, total sky and global radiation and of global illuminance. These measurements relate to cloudless sky and to solar altitudes from 15° to 60° and have been made at Karlsruhe.

Relative luminance distribution. Proceeding from his results on J the author derived a formula for computing the relative luminance distribution $\hat{j}(h, \Psi, T)$ in dependence on solar altitude h , angular distance Ψ between sun and the celestial point considered and for different intervals of the turbidity factor T ranging from $T = 3.5$ to 6.7 . This formula was obtained by adapting two parameters in Kittler's (113) relationship for $\hat{j}(h, \Psi, T)$.

Absolute luminance distribution. An approximate formula for the luminance distribution \hat{J} over the sky reads (Dietze 54):

$$\hat{J} = \hat{S}_0 \frac{m\zeta}{m - m_\zeta} [\exp(-k_R m_\zeta T) - \exp(-K_R m T)] \frac{f(\Psi)}{4\pi} \quad (119)$$

where $f(\Psi)$ designates the effective scattering function in dependence on the scattering angle Ψ . This formula is based on the assumption of single scattering, $m = \sec z$, $m_\zeta = \sec \zeta$.

From the measured values for \hat{J} Liebelt & Bodmann (55) derived the scattering function $f(\Psi, h, T)$ in dependence on scattering angle Ψ solar altitude h and turbidity factor T . The authors represent the results on $f(\Psi, h, T)$ by means of an empirically obtained numerical approximation formula (Bodmann, Burger & Liebelt 56):

$$f(\Psi, h, T) \hat{S}_0 (klx) = [5.2 + 0.7 T + \frac{52 T - 88}{h + 10}] \cdot [92 \exp(-2.6\Psi) + 13.5 - T] \quad (120)$$

where h and Ψ must be expressed in degrees and radians respectively

The distribution of the luminous efficiency over the sky.

From a large number of measured values of sky luminance $J(\text{cd m}^{-2})$ and sky radiance $J(\text{Wm}^{-2} \text{ster}^{-1})$ Liebelt (14) examined the

distribution of luminous efficiency K_J (cd W^{-1}) = \hat{J} / J over the sky. The author found that with the exception of limited angular regions near the sun and the horizon the luminous efficiency amounts on the average to $K_J = 114 \pm 9.3 \text{ lm W}^{-1}$ for all values of solar altitude ($15^\circ - 65^\circ$) and turbidity ($3 < T < 7.5$) considered. According to Liebelt the distribution of K_J can be characterized with good accuracy by considering the means \bar{K}_J relating to three angular regions

$$\text{"Sky"} \quad \bar{K}_J = 114.0 \text{ lm W}^{-1}$$

$$\text{"Sun"} \quad \Psi < 6^\circ, \quad \bar{K}_J = 109.2 \text{ lm W}^{-1}$$

$$\text{"Horizon"} \quad \zeta > 80^\circ, \quad \bar{K}_J = 85.8 \text{ lm W}^{-1}$$

(assuming average obstruction of large city or forest stand).

The distribution of sky radiance.

Liebelt determines the distribution of sky radiance J ($\text{Wm}^{-2}\text{ster}^{-1}$) from the luminous distribution \hat{J} (cd m^{-2}) by means of the average values \bar{K}_J (lm W^{-1}) corresponding to the angular regions "Sky", "Sun" and "Horizon" mentioned above:

$$J = \hat{J} / \bar{K}_J \quad (121)$$

Sky irradiance on inclined surfaces.

The sky irradiance $H[G]$ on inclined surfaces is obtained by integration of radiance J over the hemisphere. Formula (101) applies in principle. Actually the integration is more involved, because \hat{J} contains the scattering function $f(\Psi, h)$ according formula (120). Liebelt (14) uses a computer program for calculating $H[G]$ from his results on \hat{J} and \bar{K}_J . Liebelt & Bodmann (55) briefly discuss their results on a South orientated solar collector of 60° inclination.

B 8.5 Aidynli's results on the irradiance of the unclouded sky on different inclined surfaces.

Aidynli (9) considers the ratio $P_{H_1} = H_1[G] / H_1[H]$ of the irradiances of the unclouded sky on an inclined and a horizontal

surface respectively. Values of P_{H_1} based on Liebelt's results on sky radiance distribution J and relating to $3.5 < T < 6.4$ are presented in a table, which covers various solar altitudes and orientations of the receiving surface. This table contains furthermore the corresponding ratios computed from the luminance distributions by Kittler (52), Gusev (52) and Nagata (57). Larger differences between the results obtained for the various radiance and luminance distributions are found only for the cases of low solar altitude and high turbidity, combined with high values of inclination and azimuth of the surface.

B 8.6 The formulae of Temps and Coulson

Coulson et al. (51) examined the reflection and polarization characteristics of selected natural and artificial surfaces and found an angular dependence in the reflectivity of the surfaces of grass studied. Furthermore Temps & Coulson (50) investigated the irradiance from sun and sky in cloudless conditions by means of a precision type Eppley pyranometer, which could be turned into any desired direction. This instrument was installed at a height of 2 m in an open field of short grass near Davies, California. The latter authors applied the results of these measurements to check the validity of the values for $H[G]$ and $R[G]$ derived from the formulas (95) to (97) and by means of the method of computation according to Robinson & Schüepp (5). Considerable deviations between the measured and computed values of sky irradiance on inclined surfaces were found. This disagreement is mainly caused by Robinson & Schüepp's assumption of an isotropic distribution of the diffuse radiation.

$$C_1 = 1 + \frac{\sin^3 \frac{\theta}{2}}{2} \quad (122)$$

for increased radiance near the horizon

$$C_2 = 1 + \cos^2 \theta \sin^3 z \quad (123)$$

for increased radiance around the sun

$$C_3 = 1 + \sin^2 \frac{z}{2} |\cos a| \quad (124)$$

for the anisotropy of ground reflection

The principal sources of anisotropy are the increase of sky radiance towards the horizon, the circumsolar radiation and the anisotropy of the radiation reflected from the ground. Temps & Coulson derived three empirical correction factors which should take account of the three different sources of anisotropy.

The modified relations read:

$$H[G]_{\text{corr}} = H[G]_{\text{isotrop}} C_1 \cdot C_2 \quad (125)$$

$$R[G]_{\text{corr}} = R[G]_{\text{isotrop}} C_3 \quad (126)$$

Temps & Coulson emphasize that the extent to which their method is applicable to other geographic locations and atmospheric conditions is a subject which requires further studies.

The above correction factors C_1 to C_3 have been derived for the cloudless sky. In a more recent paper Klucher (58) gives empirically obtained correction factors C'_1 and C'_2 which allow the extension of the Temps & Coulson model to the clouded sky. The modified corrections are:

$$C'_1 = 1 + F \sin^3 \frac{s}{2} \quad (127)$$

$$C'_2 = (1 + F \cos^2 \theta \sin^3 z) \quad (128)$$

$$F = 1 - \left(\frac{D[H]}{G[H]} \right)^2 \quad (129)$$

B 9. SIMPLER METHODS INVOLVING AN APPROXIMATE CONSIDERATION OF THE ANISOTROPY OF SKY RADIATION

The increased radiance of circumsolar radiation has been mentioned as a source of the anisotropy of sky radiation in section 8.6. Some authors separate formally the circumsolar radiation from the sky radiation of the remaining regions of the hemisphere. Circumsolar radiation is then considered as a parallel beam of radiation coming from the direction of the sun, although its angular aperture may sometimes amount to 20° or more, depending on turbidity or kind of clouds.

The radiation $B[H]$ of the remaining regions of the sky ("background radiation") and

the radiation $R[H]$ reflected from the ground are assumed as isotropic, so that formulae (95) and (96) apply in this approximation. Under these assumptions we have:

$$H[H] = Z[N] \sin h + B[H] \quad (130)$$

for horizontal surface

$$H[G] = Z[N] \cos \theta + B[H] \cos^2 \frac{s}{2} \quad (131)$$

for inclined surfaces

If measured values $H_M[H]$ for sky radiation on a horizontal surface are available and data or formulae for one of the components $Z[N]$ or $B[H]$ are known, the values of sky irradiance $H[G]$ on any inclined surface can be determined. The methods considered in this section differ according to which of these components are given by measured data or computed from suitable parameters.

B 9.1 Loudon's method

Loudon (59) measured the background sky radiation $B[H]$ by means of a pyranometer, shading direct solar radiation and circumsolar radiation up to about 30° from the direct solar disk. The author found the $B[H]$ values to be nearly independent of cloudiness for cases when the sky is neither unclouded nor completely overcast. The results, which are given in a table for 10° to 60° solar altitude, relate to the region near London and to the cases of a unclouded and a clouded, but not overcast sky. For an overcast sky the results obtained in cloudless conditions represent the better approximation.

Loudon proceeds from measured values $G_M[H]$ of global irradiance on a horizontal surface. The relations corresponding to formulae (130) and (131) for this case:

$$G[H] = (S[N] + Z) \sin h + B[H] \quad (132)$$

$$G[G] = (S[N] + Z) \cos \theta + B[H] \cos^2 \frac{s}{2} + AG[H] \sin^2 \frac{s}{2} \quad (133)$$

Introducing relation $S[N] + Z = \frac{G[H] - B[H]}{\sin h}$, which corresponds to the approximation considered, we have for global irradiance on inclined surfaces:

$$G[G] = (G_M[H] - B[H]) \frac{\cos \theta}{\sin h} + B[H] \cos^2 \frac{s}{2} + AG[H] \sin^2 \frac{s}{2} \quad (134)$$

The diffuse irradiance $D[G]$ on inclined surfaces can be determined, if measured values $S_M[N]$ and $R_M[H]$ of direct solar intensity and the radiation reflected from the ground are available in addition to global intensity $G[H]$. Computing the sky radiation on a horizontal surface by means of relation $H_M[H] = G_M[H] - S_M[H] \sin h$ and proceeding from Loudon's values $B[H]$ for the background radiation we have:

$$D[G] = (H_M[H] - B[H]) \frac{\cos \theta}{\sin h} + B[H] \cos^2 \frac{s}{2} + R_M[H] \sin^2 \frac{s}{2} \quad (135)$$

The last term in (135) is replaced by the expression $AG_M[H] \sin^2 \frac{s}{2}$ if no measured values for $R_M[H]$ are available. If furthermore no measurements of albedo A exist for the location, an estimated value may be applied.

B 9.2 Hay's method

Hay (60) considers the ratio $Q = Z[N] / H[H]$ between the values of the intensity of circumsolar radiation $Z[N]$ and sky radiation $H[H]$ from the whole sky on a horizontal surface. Using ratio Q the circumsolar component and the remaining background component of diffuse sky radiation can be expressed as:

$$Z[N] = Q H[H] \quad (136)$$

$$B[H] = H[H] (1 - Q) \quad (137)$$

As in Loudon's approach circumsolar radiation is treated as parallel radiation coming from the direction of the sun, whereas isotropy is assumed for the background radiation. Furthermore the author's method is based on the following assumption made for Q :

$$Q = q = \left(\frac{S}{S_0} \right)^{\frac{1}{m}} \quad (138)$$

where q represents the integral transmission factor of the atmosphere for solar radiation.

The above relations (130) and (131) for sky irradiance on horizontal or inclined surfaces respectively read in this case:

$$H[H] = (Z[H] + B[H]) = qH[H] \sin h + (1-q)H[H] \quad (139)$$

$$H[G] = qH[H] \cos \theta + (1-q)H[H] \cos^2 \frac{s}{2} \quad (140)$$

Adding $R[H] = A G[H]$ for the contribution of ground reflexion we have for the whole diffuse irradiance on inclined surfaces

$$D[G] = q H[H] \cos \theta + (1 - q) H[H] \cos^2 \frac{s}{2} + A G[H] \sin^2 \frac{s}{2} \quad (141)$$

Measured data on short period values of $H[H]$, $S[N]$ or $G[H]$, $S[N]$ or $H[H]$, $G[H]$ respectively, as well as on albedo are needed as input data.

Hay's assumption (138) is valid for the case of no atmosphere, $q = 1$ and $H = B = 0$, as well as for overcast sky with $q = 0$ and $B = H$. The assumption is furthermore plausible for a cloudless turbid sky. There exist situations for which relation (138) does not apply, such as the case of a partial cloud cover and small turbidity, if the region around the sun is free of clouds.

Hay compared measured and computed hourly values of global irradiance on south orientated surfaces inclined 30° , 60° , and 90° . This comparison relates to values measured at Toronto and Vancouver. The calculated irradiances were obtained according to four different computing models characterized by the following simplifying assumptions:

- a) Sky radiance uniformly distributed over the sky hemisphere, ($Q = 0$)
- b) All the diffuse radiation originates close to the solar disc ($Q = 1$)
- c) Half of the diffuse radiation is circumsolar ($Q = 0.5$)
- d) The above relations (136) to (141) apply. Measured values for the reflected radiation $R[H]$ are used for the computations.

As a result of these comparisons the author finds that the models a) to c) can produce substantial systematic errors, whereas model d) generally yields reduced systematic and random errors in the calculated global irradiances considered.

B 9.3 Buglar's method

Buglar's (61) approach allows the conversion of measured hourly values of global irradiation on a horizontal surface into the corresponding hourly values for an inclined surface. Although the author's aim is the computation of global radiation, the principal assumptions concern the sky component. Buglar's method may therefore best be discussed in this subsection. All formulae relate to hourly values.

The procedure for cloudless sky

The direct solar intensity $S_1[N]$ is obtained from curves (as functions of atmospheric precipitable water and dust content) given by Rao & Seshadri (62). The circumsolar component is taken as being 5% of the direct solar intensity:

$$Z[N] = 0.05 S_1[N] \quad (142)$$

The background radiation $B_1[H]$ is considered as uniformly distributed and computed from the formula:

$$B_1[H] = 16.0 h^{0.5} - 0.4 h \quad (143)$$

expressed in $W_m^{-2} h^{-1}$

The hourly values $H_1[H]$ and $G_1[H]$ of sky and global irradiation can then be obtained by straightforward geometry and adding up the direct, circumsolar and background components respectively.

The procedure for the clouded sky

Measured hourly values $G_{2M}[H]$ of global radiation for arbitrary cloud conditions are given as input data. The author gives formulae to compute hourly values $B_2[H]$ of the background irradiation depending on $G_{2M}[H]$ and of the characteristic ratio Q which is defined by the formula:

$$Q = G_{2M}[H] / G_1[H] \quad (144)$$

According to Buglar the relations for the background radiation are:

$$B_2[H] = 0.94 G_{2M}[H] \quad (145a)$$

for $0 < Q \leq 0.4$

$$B_2[H] = \frac{1.29 - 1.19 Q}{1.00 - 0.334 Q} G_{2M}[H] \quad (145b)$$

for $0.4 < Q < 1.0$

$$B_2[H] = 0.15 G_{2M}[H] \quad (145c)$$

for $Q > 1.0$

The author assumes furthermore for the circumsolar component in clouded conditions:

$$Z_2[N] = \sigma Z_1[N] = 0.05\sigma S_1[N].$$

The relations for sky radiation on horizontal and inclined surfaces are in this case.

$$H_2[H] = 0.05\sigma S_1[N] \sin h + B_2[H] \quad (146)$$

$$H_2[G] = 0.05\sigma S_1[N] \cos \theta + B_2[H] \cos^2 \frac{s}{2} \quad (147)$$

Adding the well known approximate term for the reflected radiation it follows for the whole diffuse radiation on inclined surfaces:

$$D_2[G] = 0.05\sigma S_1[N] \cos \theta + B_2[H] \cos^2 \frac{s}{2} + AG_{2M}[H] \sin^2 \frac{s}{2} \quad (148)$$

According to Buglar's assumptions only the values for $S_1[N]$ (taken from Rao & Seshadri's results) and measured hourly values $G_{2M}[H]$ for global radiation are available as input data. The hourly values of relative sunshine duration σ are obtained from the expression for hourly global irradiation $G_{2M}[H]$ on a horizontal surface. The expressions for global radiation reads:

$$G_{2M}[H] = 1.05\sigma S_1[N] \sin h + B_2[H] \quad (149)$$

$$G_2[G] = 1.05\sigma S_1[N] \cos \theta + B_2[H] \cos^2 \frac{s}{2} + AG_{2M}[H] \sin^2 \frac{s}{2} \quad (150)$$

As $B_2[H]$ is determined by known input values and given relations the hourly relative sunshine duration can be obtained by solving equation (149) for σ . Inserting the latter values for σ into relations (148) and (150) the hourly values of diffuse and global irradiation are computed.

Buglar checked his method using measured hourly values of global irradiation both on a horizontal surface and on a plane inclined 38° facing North. These reference data have been measured at Melbourne over a 5-year period. The differences between the computed and measured hourly values are found to be approximately distributed about zero with a standard deviation of $\pm 0.16 \text{ MJm}^{-2}$.

B 9.4 Testing computed values of diffuse irradiance

Valko (63) compared measured values of sky irradiance on vertical and inclined surfaces with the results obtained according to various computing methods discussed in the foregoing subsections. Two sets of reference data have been chosen:

- 1) Hourly values $D_M[G]$ of diffuse irradiation on a South facing vertical plane, measured at Locarno-Monti on 1 July in 5 different years. Clear sky conditions of different turbidity and cloudy conditions are represented. These data have been gained by recording $G[G]$, $G[H]$ and $H[H]$ and computing

$$D[G] = G[H] - (G[H] - D[H]) \cos \theta / \sin h \quad (151)$$

whereby the angles h and θ relate to the middle of each hourly interval.

- 2) Instantaneous values of diffuse irradiance measured by means of a mobile equipment (64) in Carpentras, on surfaces inclined 39°, 60° and 90° and orientated towards East, South, West and North respectively. Three sets of measurements relating to different times of the day and to days of different turbidity have been chosen.

These reference data have been compared with the values computed according to the methods of Temps & Coulson (50), Loudon (59) and Hay (60) discussed in the section above. Included are furthermore values obtained by means of the isotropic model. The results of these comparisons giving the relative deviations

$$\Delta D[G] = (D_{\text{computed}} - D_{\text{measured}}) / D_{\text{measured}} \quad (152)$$

expressed in percent are summarized by the author in Table 8 and 9 of reference (63) for Locarno-Monti and Carpentras respectively. The ground albedo was assumed to be $A = 0.2$ at both sites (concrete roof).

B 10. COMPUTING GLOBAL IRRADIANCE ON INCLINED SURFACES

Combining the components of direct solar radiation, sky radiation and radiation reflected by the ground we obtain the expression for global irradiance on inclined surfaces:

$$G[G] = S \sin \theta + H[G] + R[G] \quad (153)$$

The components of direct solar irradiance and sky irradiance can be determined by one of the computing procedures discussed in the foregoing sections 7 to 9. The calculation of the direct solar term is rather straightforward in many cases. A more indirect method for determining solar intensity S_1 for the turbid but cloudless sky, based on Berlage's formula (35) and on measured hourly values $G_{LM}[H, h, m]$ of global radiation has been applied by Basnet (19) as stated in subsection 3.2. Most authors assume isotropy for the reflected radiation using the approximation:

$$R[G] = A G[H] \sin^2 \frac{s}{2} \quad (154)$$

The correction factors derived by Temps & Coulson (50) which should be applied to formula (154) and which take account of the anisotropy of ground reflection have been mentioned in subsection 8.6.

Relation (153) can be expressed in the form

$$G[G] = S[H] P_S + H[H] P_H + G[H] P_R \quad (155)$$

where P_S , P_H and P_R are the conversion factors for the direct, diffuse and ground reflected radiation respectively. Liu & Jordan's (44) procedure which is limited to South facing surfaces may be resumed here. Considering daily values of irradiation and dividing by $G[H]$ these authors put:

$$P_G[d] = G[G, d] / G[H, d] = 1 - \frac{H[H, d]}{G[H, d]} P_S[d] + \frac{H[H, d]}{G[H, d]} P_H[d] + P_R[H] \quad (156)$$

The factors P_S , P_H , P_R and the ratio $H[H,d] / G[H,d]$ are discussed separately. The approximate computation procedure for factor P_S , valid for southfacing inclined surfaces has been described in subsections 7.3 and 7.4 above.

An isotropic distribution of sky radiance is assumed. According to relation (95) in subsection 8.1 the formula for factor $P_H[d]$ reads:

$$P_H[d] = \frac{1}{2} (1 + \cos s) = \cos^2 \frac{s}{2} \quad (157)$$

The ground surrounding the receiving surface is considered as consisting of a number of areas with differing values of albedo A_i . The radiation reflected from each area is assumed as isotropically distributed. A formal expression defining an effective value of albedo A_{eff} for this case is given. In accordance with relation (96) and (97) the formula for factor P_R becomes:

$$P_R[d] = \frac{1}{2} A_{eff} (1 - \cos s) = A_{eff} \sin^2 \frac{s}{2} \quad (158)$$

Liu & Jordan (44,45) established a statistical relationship between the ratio $H[H,d] / G[H,d]$ on the one hand and the characteristic ratio $Q = G[H,d] / G_0[H,d]$ on the other hand, as has been shortly discussed in subsection 6.1 and expressed by formulae (64a) and (64b) in Table 2. The authors apply this relationship to determine the values of the ratio $H[H,d] / G[H,d]$ in formula (156) above.

Finally the total conversion factor $R_G[d]$ is computed by (156) and the daily global irradiation on the inclined surface.

$$G[G,d] = P_G[G,d] G[H,d] \quad (156a)$$

is obtained from the corresponding values for a horizontal surface.

Whereas Liu & Jordan's approximate computing procedure for $P_S[d]$ works for South facing inclined surfaces only, the formulae (157) and (158) are valid within the limits of the isotropic model for surface of any orientation.

As mentioned in section 7.4, Klein (49) has extended the Liu & Jordan model to surfaces of arbitrary orientation. The general relation (156) is used also in Klein's model, but more involved formulae, described in subsection 7.4 apply for the factor $P_S[d]$. The formulae (157) and (158) for the remaining factors are the same as in Liu & Jordan's model. Considering this adaptation the relations (156) are valid, within the limits of the approximation, for any surface orientation. In his paper Klein compares value of the factor P_G derived from measurements with the corresponding values computed according to Liu & Jordan's method. An albedo of $A_{eff} = 0,2$ is assumed

The results presented in METEOPLAN

In METEOPLAN: "Solar Radiation Impact on Buildings of Different Shape and Orientation", containing 975 graphs, Valko (70) gives a comprehensive representation of direct, diffuse and global irradiance on rectangular prisms of arbitrary shape for the cloudless sky. Also included are, among others, numerical tables for the basic irradiation values $S_1[N]$, $D_1[H]$ and D_1 [vertical surface], as well as direct, diffuse and global irradiance on vertical cylinders. The results are presented for different values of the turbidity coefficient, which vary in steps of $\Delta\beta = 0.050$ from $\beta = 0.050$ to 0.250. No computing models were adopted. The basic relationships were determined by evaluating statistical data obtained by measurements extended over many years (65, 71). Particular care was taken when deriving the diffuse radiation component. All radiation data presented are of general applicability. The practical use of the many tables and charts is explained in the introductory part of the work.

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APPENDIX C (CHAPTER 1)

MODELS FOR ESTIMATING INCOMING SOLAR IRRADIANCE

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LIST OF SYMBOLS

CO	total cloud opacity	H'	half day length
CA	total cloud amount	I	direct beam irradiance
C _e	effective cloud amount	I	radiant intensity
C _i	cloud amount in the ith layer	I(0)	solar constant (spectrally integrated direct beam irradiance at the top of the atmosphere normal to the Sun) corrected for the instantaneous departure of the actual Sun-Earth distance from the mean value.
C _i '	observed cloud amount in the ith layer	I(0)	extraterrestrial irradiance = I(0)μ ₀
C _s	sum of non-cirrifiform cloud amount	I ₀	cloudless sky direct beam irradiance
C*	reported amount of cirrus cloud	I _x	direct beam irradiance uncorrected for attenuation by cirrus
D	diffuse irradiance	LAT	local apparent time
D'	diffuse irradiance without the effect of multiple reflections	LS	longitude of a station
D _A	diffuse irradiance due to aerosol scattering	LSM	longitude of the standard meridian
D _R	diffuse irradiance due to molecular (Rayleigh) scattering	LST	local standard time
D _S	diffuse irradiance due to multiple reflections between ground and atmosphere	MBE	mean bias error
D ₀	cloudless sky diffuse irradiance	N	potential number of sunshine hours
ET	equation of time	P(μ', φ', μ, φ)	phase function to account for scattering of an incident ray from the direction μ', φ' into the direction μ, φ
F	measured irradiance	Q _{ext}	Mie extinction efficiency
F'	calculated irradiance	RMSE	root mean square error
G	global irradiance	R*	actual Sun-Earth distance
G'	global irradiance without the effect of multiple reflections	—	
G(0)	daily total extraterrestrial irradiance	R*	mean Sun-Earth distance
G ₀	global irradiance for cloudless skies	T _r (a)	transmittance after extinction by aerosols
H	hour angle		
H _p	scale height		

$T_R'(a)$	$T_R(a)$ at $m_r = 1.66$	s	hourly fraction of bright sunshine
$T_R(o)$	transmittance after absorption by carbon dioxide and ozone	t_c	cloud transmittance
$T_R(R)$	transmittance after scattering by dry air molecules	t_i	cloud transmittance in the i th layer
$T_R(t)$	total cloudless sky transmittance	t^*	transmittance of cirriform cloud
$T_R(w)$	transmittance after absorption by water vapour	u_0	equivalent depth of ozone in the atmosphere
T_d	dew point temperature	u_w	corrected water path length
T_o	standard surface temperature, 273°K	u_w'	uncorrected water path length
T_s	surface temperature	z	height
V	visibility (km)	z_1	height of lowest cloud
a_c	cloud absorptance	z_T	height of top of the atmosphere
a_o	ozone absorptance	α_b	albedo of the atmosphere for surface reflected radiation
a_w	water vapour absorptance	$\alpha_b(a)$	aerosol component of the atmospheric albedo
d_f	Won's dust factor	$\alpha_b(c)$	cloud component of the atmospheric albedo
d_n	day number	$\alpha_b(R)$	Rayleigh scattering component of the atmospheric albedo
f	ratio of forward to total scatter	α_c	cloud albedo
f'	f at $m_r = 1.66$	α_s	surface albedo
k	aerosol transmission parameter	$\alpha_s(H)$	surface albedo at upper temperature threshold $T(H)$
k_λ	spectral mass absorption coefficient	$\alpha_s(L)$	surface albedo at lower temperature threshold $T(L)$
m_r	relative optical air mass	α_{cs}	albedo of non-cirriform cloud
n	measured number of sunshine hours	α^*	albedo for cirrus clouds
$n(r)$	number density of a particle size distribution	β	backscatterance for blue sky
p	station pressure	β_0	total scattering coefficient
p_0	standard sea level pressure, 101.3 kPa	δ	solar declination
r	particle radius		

η	complex index of refraction
θ	zenith angle
θ_0	$2\pi d_n/365$
λ	wavelength
μ	cosine of the zenith angle
μ_0	cosine of the solar zenith angle
σ_λ	spectral mass scattering coefficient
τ	atmospheric optical depth
$\tau_{a\lambda}$	spectral optical depth due to absorption and scattering (extinction) by particulates
$\tau_{g\lambda}$	spectral optical depth due to absorption by gases
$\tau_{R\lambda}$	spectral optical depth due to molecular (Rayleigh) scattering
$\tau_{w\lambda}$	spectral optical depth due to absorption by water vapour
ϕ	azimuth angle
ξ	station latitude
ψ_c	cloud transmittance
ψ_i	transmittance for the i th cloud layer
ψ_0	total cloud field transmittance for direct beam irradiance
ω_0	single scattering albedo

C 1.0 REVIEW AND CLASSIFICATION OF SOLAR RADIATION MODELS

C 1.1 Introduction

The energy crisis has prompted a dramatic increase in research associated with solar energy utilization. In this connection there is a need to provide spatial and temporal data on the solar irradiance and its direct beam and diffuse components since these define the climatological potential for utilization. Meteorological networks are poorly prepared to provide such data. Although Canada's radiation network is exceptional compared with the networks of most other countries it is still sparse especially for the direct beam and diffuse components. Therefore, there is a need for calculation procedures which can be used to provide estimates for places where measurements are not made and for places where there are gaps in the measurement record.

Many procedures can be found in the meteorological and engineering literature. Most were developed to satisfy a particular, local need and should not be considered as general models with universal application. Regression-based models generally fall into this category and care should be exercised in applying them beyond the domain for which they were derived. However, the overall forms of these models may be universally applicable so that the user needs only to verify or revise the numerical values of constants and coefficients for his particular location and time period. Since measurements are needed for this, such calibration of a model is somewhat self defeating. In tackling the task of providing a classification of solar radiation models which can serve as the basis for developing a handbook of such models the stand was taken that models and model forms which either make some claim to generality, or may be of general application, should be emphasized.

C 1.2 Radiation Transfer and the Modelling Problem

Solar irradiance at the ground is a function of the irradiance at the top of the

atmosphere and absorption and scattering by atmospheric constituents. The transfer process is described by

$$\frac{\mu}{4\pi} \frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda(\tau_\lambda) \frac{-\omega_{0\lambda}}{4\pi} \int_0^1 \int_0^{2\pi} I_\lambda(\tau_\lambda, \mu', \phi') \cdot P_\lambda(\tau_\lambda; \mu', \phi' \rightarrow \mu, \phi) d\mu' d\phi' \quad (1.1)$$

where I_λ is the spectral radiant intensity ($Wm^{-2}sr^{-1}\mu m^{-1}$) at wavelength λ ; μ the cosine of the zenith angle θ of a ray; ϕ the azimuth angle of the ray; τ_λ the atmospheric optical depth; $p(\tau_\lambda; \mu', \phi' \rightarrow \mu, \phi)$ the phase function to account for scattering of an incident ray from the direction μ', ϕ' into the direction μ, ϕ ; and $\omega_{0\lambda}$ is the single scattering albedo. If k_λ and σ_λ are the mass absorption and scattering coefficients, optical depth and single scattering albedo are defined by

$$\tau_\lambda = \int_0^z (k_\lambda + \sigma_\lambda) dz \quad (1.2)$$

and

$$\omega_{0\lambda} = \sigma_\lambda / (k_\lambda + \sigma_\lambda), \quad (1.3)$$

where z is height. The integral term in (1.1) incorporates multiple scattering in the atmosphere which is important within clouds and aerosol. The treatment of this term poses the greatest problem in solving (1.1). Solar irradiance at the ground is obtained by solving this integro-differential equation in τ_λ and integrating over azimuth and zenith angles to obtain spectral irradiance values. Total irradiance requires a further integration over the wavelength range of the solar spectrum. Analytic solutions to (1.1) are only obtained for ideal atmospheres where a simple phase function applies. In real atmospheres where aerosols and clouds are important, scattering is strongly anisotropic and the phase function must be approximated numerically. This is often done using a Legendre polynomial expansion (Chandrasekhar, 1960). The most exact solutions place extensive demands on computer time (Braslau and Dave, 1973). Faster, parameterized methods (e.g. Leighton, 1980) require detailed specification of atmospheric constituents which is rarely possible for real atmospheres.

In the case of direct beam radiation the phase function can be neglected and the transfer equation can be integrated directly to give Beer's law, i.e.

$$I_{\lambda}(\tau_{\lambda}) = I_{\lambda}(0) \exp(-\tau_{\lambda}/\mu_0) \quad (1.4)$$

where $I_{\lambda}(0)$ is the spectral intensity at the top of the atmosphere ($\tau_{\lambda} = 0$) and μ_0 is the cosine of the solar zenith angle. The optical depth can be written as:

$$\tau_{\lambda} = \tau_{g_{\lambda}} + \tau_{w_{\lambda}} + \tau_{R_{\lambda}} + \tau_{a_{\lambda}} \quad (1.5)$$

where $\tau_{g_{\lambda}}$, $\tau_{w_{\lambda}}$, $\tau_{R_{\lambda}}$ and $\tau_{a_{\lambda}}$ are the component optical depths due to absorption by gases and water vapour, scattering by dry air molecules (Rayleigh scattering) and extinction (absorption and scattering) by particulates. Braslau and Dave (1973) have applied (1.1) to 80 wavebands across the solar spectrum for several model atmospheres with fixed gaseous and aerosol contents. For real atmospheres there are uncertainties in the distribution of gases, the spectral values of absorption coefficients and the quantities, species and optical properties of particulates. Aerosol effects are difficult to include. Usually the Mie theory (Van de Hulst, 1957) is used to evaluate $\tau_{a_{\lambda}}$ from

$$\tau_{a_{\lambda}} = \int_0^{z_T} dz \int_0^{\infty} \pi r^2 Q_{\text{ext}}(2\pi r/\lambda, \eta) n(r) dr \quad (1.6)$$

where Q_{ext} is the Mie extinction efficiency for a particle radius r and complex index of refraction η , $n(r)$ is the number density of a particle size distribution at radius r , and z_T is the height of the top of the atmosphere. Optical depths due to either absorption or scattering by particulates can be calculated from expressions analogous to (1.6) with appropriate Mie absorption or scattering efficiency factors. Efficiency factors are calculated from available subroutines (Dave, 1968). Partitioning of extinction between absorption and scattering can only be attempted if the refractive index can be specified. If it is known, together with the particle size distribution at different heights, $\tau_{a_{\lambda}}$ can be evaluated from (1.6). The large amount of calculation precludes routine application of (1.6) to radiation models which must be applied in a

large number of calculations at one or more locations.

Clouds present similar problems to aerosol. In principle, radiation transfer through clouds can be treated in exactly the same way as transfer through aerosols. The size distribution and optical properties are different but the Mie theory is still applicable. Clouds, however, pose additional problems. They develop and disperse very rapidly, occur at different levels in the atmosphere and have different structures. There is usually very little such information for clouds so that they are poorly specified. At best, only statistical properties of transmission, absorption and reflection can be assigned and at any given instant actual properties may differ substantially from the statistical values that a model uses. This uncertainty is amplified by inadequate data on cloud amount, particularly within the various layers in which cloud occurs. Upper cloud layers are often obscured by low cloud so that the ground-based observer can not identify their type and amount. Satellites, likewise, are often unable to see lower layers below substantial amounts of high cloud. These uncertainties concerning cloud pose the greatest problems for radiation modelling. In the absence of cloud and large amounts of aerosol the instantaneous amount of radiation reaching the ground can be calculated to an accuracy of $\pm 5\%$ (Robinson, 1970). In the presence of cloud there is no model that can attain such accuracy instantaneously although it can be attained over longer time periods.

More formal solutions to the transfer equation include the doubling method (Hansen, 1971), method of discrete ordinates (Chandrasekhar, 1960), series solutions (Braslau and Dave, 1973), and the method of spherical harmonics (Bergstrom and Viskanta, 1972). A detailed discussion of these and other methods can be found in Lenoble (1977). Generally, they have been applied to model atmospheres and are not used to calculate irradiances routinely. Instead of the formal solutions, approximations with varying degrees of empiricism have been employed. Some have maintained explicit links with the radiative transfer equation while the link is tenuous in others.

C 1.3 Classification of Solar Radiation

Models

A broad classification will be presented which encompasses most types of models and emphasizes interrelationships between models. There is a model hierarchy but all stem from a common source, the radiative transfer equation.

It is appropriate at this point to identify criteria which are important in designing or selecting a solar radiation model. In general a model

1. can only require readily available meteorological data which are directly measured or observed at meteorological stations,
2. must provide irradiance estimates to an acceptable level of accuracy over a useful time period,
3. must be economical in terms of computation.

Specification of the second and third criteria will depend on the nature of the study. If a computer is to be used more sophisticated models involving detailed calculations and large amounts of data can be handled. Without such computation facilities only the simplest of models requiring small amounts of data are feasible.

Formal solutions to the transfer equation calculate spectral values of irradiance and deal explicitly with multiple scattering through some solution of the phase function integral. This procedure is generally too detailed for routine application and it is debatable whether our knowledge of atmospheric constituents at any place and time is sufficient to justify such an approach (Lacis and Hansen, 1974; Atwater and Ball, 1978; Bergstrom and Petersen, 1976). It is not generally used for calculating solar irradiance at the ground.

All the models considered in this study are non-spectral and deal with single scattering alone although several now include the effect of multiple reflection between the ground and the atmosphere. Invariably the atmosphere is treated as plane parallel.

The most physically-based models may be termed "Layer Models" and include those developed at the Center for Environment and Man - CEM (Atwater and Brown, 1974; Atwater and Ball, 1978), McMaster University-MAC (Davies et al. 1975; Davies and Hay, 1980) and the University of British Columbia - CLS (Suckling and Hay, 1976, 1977). Their general form for global irradiance G is

$$G = G_0 \prod_{i=1}^n \Psi_i f(\alpha) \quad (1.7)$$

where G_0 is the cloudless sky irradiance, Ψ_i is the i th cloud layer transmittance and $f(\alpha)$ is a function of surface albedo α to incorporate multiple reflections between ground and atmosphere. Direct beam irradiance on a horizontal surface can be stated

$$I = I_0 \Psi_0 \quad (1.8)$$

where I_0 is the cloudless sky irradiance and Ψ_0 the total cloud field transmittance for direct beam irradiance. The three versions of this model use different parameterizations for transmittances. Diffuse irradiance is calculated as the difference between global and direct beam irradiances in the CEM and MAC models but is calculated independently for the CLS model as shown later.

The form of models which utilize total cloud amount CA or fractional sunshine n/N can be derived from (1.7). In the first case

$$G = G_0 \Psi_c f(\alpha) \quad (1.9)$$

where Ψ_c is the cloud transmittance which may be defined as a statistical function of cloud amount (Won, 1977; Kimura and Stephenson, 1969) or as

$$\Psi_c = (1 - CA) + t_c CA \quad (1.10)$$

in which t_c is an empirically determined cloud transmittance, a statistical average for all cloud types and amounts (Monteith, 1962; Hay, 1970). Let

$$CA = 1 - n/N \quad (1.11)$$

where n is the measured number of sunshine hours and N is the potential number (daylight hours). Then

$$\Psi_C = t_c + (1-t_c) n/N = a + b n/N \quad (1.12)$$

This sunshine-based equation is attributed to Kimball (1919) and Ångström (1924).

Equation (1.9) is often simplified further by replacing cloudless sky global irradiance with the extraterrestrial irradiance. This precludes detailed, explicit calculations of absorption and scattering in the atmosphere. Instead, these processes are accommodated statistically. The function Ψ_C is determined by regressing G/G_0 on CA or n/N . Therefore, values for the regression parameter are determined by the transmittance properties of all atmospheric constituents, not just clouds.

Equations (1.9) and (1.12) are prototypes for most models published in the last 60 years. Many have their own characteristics, parameterization and simplifications. Most were intended for estimating irradiance over daily, or longer periods. Relatively few are well suited or meant to provide hourly values. Of the broad model types considered the layer models are probably best suited in principle for this purpose since they are the most sensitive to changes in layer cloud amounts and allow cloud transmittance to vary with cloud type. Because of difficulties arising from specifying cloud cover, however, little success has been achieved for hourly estimates. Usually, weekly estimates compare with measurements to within $\pm 10\%$, which is sufficient accuracy for most purposes.

C 1.4 Representative Classification of Models

Although (1.7), (1.9) and (1.12) provide a unifying framework for radiation models the question of classification was also approached from a more empirical viewpoint. Major journals for the 1970-1980 period were searched systematically for papers concerned with solar radiation models. Earlier work was identified from references in these papers. This search produced the list of references given in this appendix.

The list is selective rather than comprehensive. The selection criteria were

1. Models must be either general or have pre-determined values for empirical components. Otherwise models are of limited usefulness. Regression models are often site-specific and can only be applied generally if numerical values of coefficients and constants have been verified or determined for the site of application.
2. Input data must be readily available.

Models were classified into five groups, four of which can be anticipated from (1.7), (1.9) and (1.12). The fifth is entirely empirical and is based on the work of Liu and Jordan (1960). The five groups are:

1. Cloudless sky models
2. Cloud layer based models
3. Total cloud based models
4. Sunshine based models
5. Liu and Jordan models.

Each group will be reviewed individually and models will be selected from each for testing against measured records at six Canadian stations over a ten-year period.

C 1.4.1 Cloudless Sky Models

These either calculate direct beam and diffuse components separately or the global irradiance alone. Rigorous application of the radiative transfer equation would involve spectral evaluation of irradiances, then integration of the results over the wavelengths of the solar spectrum. This task is seldom attempted.

(a) Direct Beam

In lieu of spectral calculations, spectrally-integrated values of direct beam irradiance are forced into the same form as (1.4) such that

$$I = I(o) \exp(-\tau/\mu_o) = I(o)T_I(t) \quad (1.13)$$

where I and $I(o)$ are spectrally integrated counterparts of $I_\lambda(\tau_\lambda)$ and $I_\lambda(o)$ but refer to a horizontal surface. The exponen-

tial term defines a transmittance $T_r(t)$ which has components for absorption by gases, usually water vapour $T_r(w)$, carbon dioxide and ozone $T_r(o)$, scattering by molecules $T_r(R)$, and extinction by aerosols $T_r(a)$. These transmittances, determined from spectral calculations (Moon, 1940; Houghton, 1954; Brooks, 1959; Lacis and Hansen, 1974; Hottel, 1975; King and Buckius, 1979) are given as functions of measurable quantities, usually, surface pressure, dew point temperature and a measure of atmospheric turbidity such as visibility.

Transmittances have often been applied multiplicatively (Houghton, 1954; Brooks, 1959; Monteith, 1962; Nunez, Davies and Robinson, 1971; Davies, Schertzer and Nunez, 1975; Suckling and Hay, 1977) so that

$$I = I(o)T_r(o)T_r(w)T_r(R)T_r(a) \quad (1.14)$$

The effect of ozone has been frequently ignored (Brooks, 1959) or set equal to about 3% of the solar constant (Unsworth and Monteith, 1972). Some studies have used Houghton's transmittance curves which include a transmittance due to water vapour scattering. Most modern work, however, does not include it. Since water vapour absorbs at longer wavelengths than ozone it is not strictly correct to apply $T_r(w)$ multiplicatively. Instead, water vapour absorptance a_w should be subtracted (Paltridge and Platt, 1976) so that

$$I = I(o)[T_r(o)T_r(R)-a_w]T_r(a) \quad (1.15)$$

However, the difference in values of direct beam irradiance calculated from (1.14) and (1.15) is less than 2%. These equations are measurable quantities which can permit predictions of good accuracy at specific locations (King and Buckius, 1979). The formulation given by ASHRAE (1972, 1975), identical in form to (1.13), does not utilize measurable quantities, but uses prescribed, climatological average values of atmospheric transmittance. It is, therefore, insensitive to weather changes. A recent study using Canadian data (MacLaren et al., 1980) showed that it performed relatively poorly.

In this study attention will be concerned on the type of formulations given in

(1.15) which use the parameterization of Lacis and Hansen (1974) or King and Buckius (1979). The first group, used in the CEM and MAC models, were developed for the Goddard Institute for Space Studies (GISS) General Circulation Model. It is also used in the General Circulation Models developed by the Atmospheric Environment Service and the National Center for Atmospheric Research. Since the parameterization incorporates modern results it is probably superior to earlier parameterizations which were based on Smithsonian studies many years ago. Tests with it have been very satisfactory (Davies and Uboegbulam, 1979; MacLaren et al., 1980). On this basis it has been used, where feasible, in all model circulations carried out in this study. The King and Buckius (1979) parameterization is more recent, with no published test at the time of writing. This study provides an opportunity to do this.

The main uncertainty in computing beam irradiance from either (1.14) or (1.15) rests in defining appropriate values for $T_r(a)$.

(b) Diffuse

Possibly because the diffuse component is less than 20% of the global irradiance on cloudless days, fewer attempts have been made to model it separately. The ASHRAE model makes it proportional to the direct beam using predetermined values for the proportionality constant. In the MAC model diffuse irradiance follows logically from (1.15). It is assumed that scattering is isotropic within a given hemisphere and that water vapour only absorbs the direct beam (Paltridge, 1973). The diffuse irradiance on a horizontal surface is the sum of components due to molecular D_R and aerosol D_A scattering:

$$D = I(o)\mu_o T_r(o) [1-T_r(R)]/2 \quad (1.16)$$

$$D = I(o)\mu_o [T_r(o)T_r(R)-a_w] [1-T_r(a)]\omega_o f \quad (1.17)$$

where f is the ratio of forward to total scatter. Because of the isotropic assumption half of the radiation scattered by molecules reaches the ground. In (1.17) the single scattering albedo allocates to scattering the fraction of the radiation attenuated by aero-

sol. Diffuse radiation is further enhanced by multiple reflections between the ground and atmosphere. The enhancement is particularly important for atmospheres overlaying surfaces with high albedo. This effect can be expressed in terms of surface albedo α_s , the albedo of atmosphere α_b for surface reflected radiation and the primary incident irradiances $(I + D_R + D_A)$:

$$D = \alpha_b \alpha_s (I + D_R + D_A) / (1 - \alpha_b \alpha_s). \quad (1.18)$$

The total diffuse irradiance is the sum of D_R , D_A and D_S .

(c) Global

The CEM model (Atwater and Brown, 1974) has used an equation similar to (1.15) to calculate global irradiance directly. The diffuse component is then obtained as a residual. In the GISS model Lacis and Hansen (1974) calculate global irradiance directly. They divided the solar spectrum into two parts at $\lambda = 0.9 \mu\text{m}$. Absorption by water vapour is confined to wavelengths greater than $0.9 \mu\text{m}$, which contain 35% of the total irradiance. At other wavelengths only the effects of molecular scattering and absorption by ozone need be considered.

Regression models which predict global irradiance as a function of cloud amount or sunshine provide estimates of cloudless sky irradiance when either $n/N = 1$ or $CA = 0$. These estimates, however, are climatological averages for the range of data that were used in determining values of the regression parameters.

C 1.4.2 Cloud Layer-based Models

These models have used either the Lacis and Hansen (1974) or Houghton (1954) parameterizations to calculate cloudless sky values of global irradiance from the procedures discussed in the previous section. Cloud effects are incorporated into these models in a very imprecise manner due to the scanty information available, once-hourly, on cloud amounts and types and the considerable variance in cloud reflectivity, absorptivity and transmissivity even within one cloud type.

The CEM, CLS and MAC models generally use the same procedure for dealing with cloud. The transmittance of a single cloud layer is given by

$$\psi_i = (1 - C_i) + t_i C_i \quad (1.19)$$

where $1 - C_i$ is the layer transmittance of the cloudfree portion of the sky at that level through which direct beam and diffuse radiation passes unimpeded and $t_i C_i$ is the cloud transmittance. Total cloud transmittance for all layers is the product of all layer transmittances (1.7). Clouds also significantly enhance the effect of multiple reflections between the ground and atmosphere. It is advisable to include this effect always. When cloud is present the multiple reflection term used to calculate diffuse irradiance under cloudless skies should be omitted. Under n layers of cloud the solar irradiance at the ground is given by

$$G = G_o \prod_{i=1}^n [(1 - C_i) + t_i C_i] / (1 - \alpha_s \alpha_b) \quad (1.20)$$

where α_b includes the effect of cloud base albedo. The CEM, CLS and MAC models employ the values of cloud transmittance given by Haurwitz (1948). These values have not been updated but there is little evidence from the work of those who have used these layer models that systematic errors arise because the Haurwitz transmission values are incorrect.

The model has been applied widely in North America and recently it has been tested in the tropical Atlantic (Davies and Uboegbulam, 1979). Layer models have not been used extensively in calculating direct beam and diffuse components. All three were tested in the MacLaren study. The results can be summarized as follows:

1. All three models performed similarly for global irradiance with the best result, in general, obtained with the MAC model with aerosol effects ignored.
2. Although, logically, from (1.20), direct beam irradiance can be stated as

$$I = I_0 \prod_{i=1}^n (1 - C_i) \quad (1.21)$$

calculated values did not compare well with measurements. Significantly better results were obtained from

$$I = I_0 (1 - CO) \quad (1.22)$$

where CO is the total cloud opacity. Better results from equation (1.22) are probably due to the better measure of total cloud amount. Although the total cloud estimate in (1.21) follows theoretically from (1.20) estimates of layer cloud amounts are difficult and less accurate than estimates of total cloud opacity. Opacity gave better results than total cloud amount probably because the opacity estimate tends to counteract the tendency of an observer to overestimate cloud amount towards the horizon.

3. Diffuse irradiance for the CEM and MAC models was calculated as the difference between the global and direct beam irradiances. The CLS procedure is slightly different permitting the diffuse component to be calculated independently. There was, however, little difference between values from the residual approach and the CLS method.

4. In the case of Toronto, the largest urban site used in the test, the effect of neglecting aerosol was noticeable.

From these findings this study proposes to re-examine the MAC and CLS models with and without aerosol estimates and using the Lacis and Hansen (1974) parameterization for both. In addition, the consequences of the change in cloud reporting initiated in 1977 by AES will be discussed. Instead of reporting actual cloud layer amounts, amounts are now only indicated as scattered, broken or overcast. However, opacity values are still reported for each cloud layer and these may be adequate replacements for cloud amounts. The suitability of layer opacities for use in these models has not been examined prior to this study.

C 1.4.3 Total Cloud-based Models

These models use only total cloud amount to define cloud transmittance as shown in (1.9). Monteith (1962) and Hay (1970) used a physically-based function (1.10) which follows logically from the layer model formulations. Values of cloud absorptivity a_c and reflectivity α_c must be specified to calculate cloud transmittance from

$$t = 1 - a_c - \alpha_c \quad (1.23)$$

Monteith (1962) assigned fixed values to a_c and α_c for the British Isles and evaluated monthly irradiances from (1.9). Hay (1970) employed values of cloud reflectivities and absorptivities given for different cloud types by London (1957) to determine mean monthly values of a_c and α_c for Canadian stations using monthly mean cloud amount and found that the calculated irradiances were within the limits of instrumental accuracy for at least 60% of the time. This model was not tested over short time periods. Hoyt (1978) and Lettau and Lettau (1969) have formulated similar models for global irradiance which utilize total cloud amount but neither provide significant advantages over the Monteith-Hay form.

Kimura and Stephenson (1969) expressed total cloud transmittance as a quadratic function of cloud opacity

$$\psi_c = a + bCO + cCO^2 \quad (1.24)$$

employing data from Ottawa alone to obtain numerical values for the constant and coefficients. Their results have been applied universally in the ASHRAE model using total cloud amount instead of opacity when the latter is not observed. Won (1977), however, has shown that values of a, b and c vary substantially with season and location in Canada. Both the ASHRAE and WON models were included in the MacLaren study. The results showed that the WON model was superior to the ASHRAE model and performed almost as well as the MAC model. However, it should be noted that it was tested with data which were used originally to provide values for a, b and c so that the evaluation was not independent.

Simplified forms of (1.9) where theoretical cloudless sky irradiance is replaced with extraterrestrial irradiance $G(0)$, also employ empirical transmittance functions obtained statistically. These functions range from simple linear form to quadratic and other non-linear functions of cloud amount (Black, 1956; Laevastu, 1960). Only the crudest estimates of irradiance can be expected from these formulae since the empirically - determined constants and coefficients for the transmittance functions must allow for transmittance through both cloud and all atmospheric constituents. There is, therefore, no possibility for accommodating the effects of short-term variations in such important quantities as atmospheric water content. At best, these formulae should only be expected to provide satisfactory estimates in the longterm. They do not estimate direct beam or diffuse components.

C 1.4.4 Sunshine-based Models

The original form proposed by Angström (1924), which consists of (1.9) and (1.12), is not widely used although it forms the foundation for the recent model of Barbaro et al. (1979). It is superior (Driedger and Catchpole, 1970) to the more common simplified version where extraterrestrial irradiance is used instead of theoretical cloudless sky irradiance.

Values of a and b in (1.12), which are obtained from regression of $G/G(0)$ against n/N , fluctuate from station to station with season (Mateer, 1955; Davies, 1965). This has meant that a and b values must be determined for a location before the formula can be applied. Rietveld (1978) has examined the variation of a and b throughout the world and found that they varied with n/N in a predictable manner. Although Rietveld (1978) has not claimed general validity for his model, his results imply this possibility.

C 1.4.5 Liu and Jordan Models

Liu and Jordan (1960) demonstrated a simple relationship between daily values of the ratio of diffuse to global irradiance and

the ratio of global to extraterrestrial irradiance using data for Blue Hill, Massachusetts. The method provides the attractive possibility of calculating diffuse and direct beam irradiance relatively simply for stations which already measure global irradiance. Ruth and Chant (1976) showed that data for four Canadian cities departed significantly from the original Liu and Jordan relationship. Hay (1976) also demonstrated that the original relationship was inappropriate in Canada and modified it by allowing for the effects of multiple reflection between ground and atmosphere. Orgill and Hollands (1977) applied the Liu and Jordan method to hourly data for Toronto and obtained good agreement with the results of Ruth and Chant (1976). On the basis of this agreement they expect their formula to apply to latitudes between 43°N and 54°N , the latitudinal range of the stations used by Ruth and Chant (1976). Collares-Pereira and Rabl (1979) using data from ten stations in the United States confirmed the result of Ruth and Chant.

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A P P E N D I X D (CHAPTER 3)

The following validation results are contained within the report, Define, Develop and Establish a Merged Solar and Meteorological Computer Data Base, MacLaren, et al. (1979). Six Canadian models are validated, four of which are referred to as "Layer" models, predominantly physical in principle. The remaining two are generally statistical, although some physical processes are utilized. The four physical models are:

- (1) CLS - (Suckling and Hay, 1977) - a cloud layer, sunshine model to calculate hourly values of beam, diffuse and global irradiance;
- (2) MAC2 and (3) MAC3- (Davies, Schertzer and Nunes, 1975) - forerunners of the CLS model; and
- (4) CEM - (Atwater and Brown, 1974) - similar to the MAC2 and MAC3 with the main differences in the parameterization of Rayleigh scattering and absorption by permanent gases and in the detail of treating multiple reflections between ground and atmosphere.

The two statistical models studied are:

- (1) WON - (Won, 1977) - a semi-empirical method using generated cloud effect regressions coefficients and transmittance parameters to compute hourly global irradiance. Slight modifications to the model enabled the calculation of diffuse and beam irradiance; and
- (2) ASH - (ASHRAE, 1975) - a model using statistical parameters of cloud effect, clearness (aerosol), and diffuse radiation.

For further information on the models consult the respective references.

Although many statistics were generated in the model validation, two of the primary ones are given in this appendix. Data from six Canadian stations were selected and the Mean Bias Error (MBE) and the Root Mean

Squared Error (RMSE) computed for each model for each station. Only the validation for global irradiance on a horizontal surface has been shown here.

TABLE D 3.1

MEAN BIAS ERROR % OF MEASURED MEAN (Hourly Data)

Station	CLS	MAC 2	MAC 3	CEM	WON	ASH
Charlottetown	1	- 4	2	2	- 4	10
Goose	2	- 3	1	2	- 1	- 9
Toronto	6	0	3	5	- 3	- 2
Winnipeg	2	- 3	0	2	- 1	- 6
Port Hardy	- 3	- 8	- 6	- 4	1	- 8
Resolute	-20	-16	-12	-11	-11	-27

TABLE D 3.2

ROOT MEAN SQUARED ERROR % OF MEASURED MEAN (Hourly/Daily)

Station	CLS	MAC 2	MAC 3	CEM	WON	ASH
Charlottetown	23/13	28/17	29/17	29/17	26/16	30/19
Goose	27/16	32/19	32/19	32/14	29/17	31/19
Toronto	24/15	28/16	28/17	28/17	26/15	26/15
Winnipeg	19/12	21/13	21/12	22/13	19/11	19/11
Port Hardy	32/19	42/26	42/25	40/25	34/19	38/24
Resolute	41/32	38/29	37/28	36/27	24/18	43/37

In addition, Figures D3.1, D3.2, D3.3, D3.4, D3.5 and D3.6 display, graphically, the monthly variation in MBE of each model at each location.

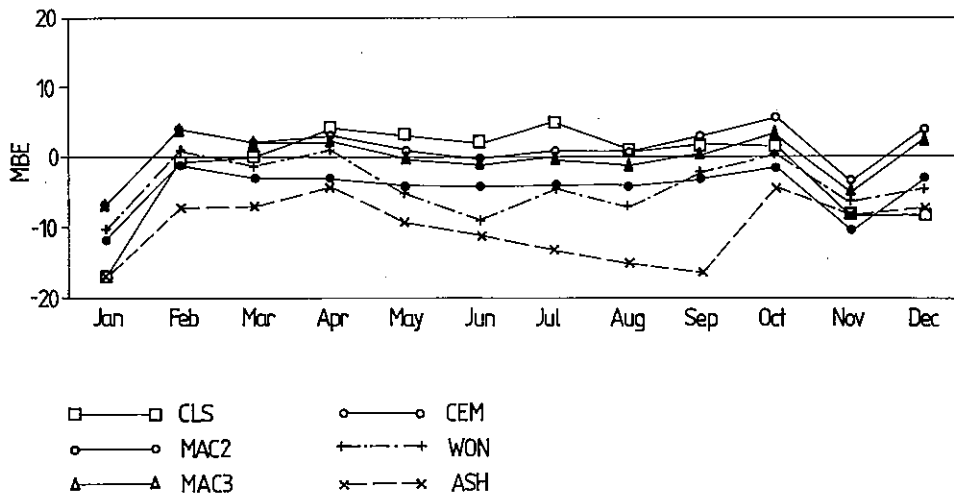


Figure D3.1
 Monthly Total MBE (% of Actual) for Global
 Radiation for 1976 at Charlottetown

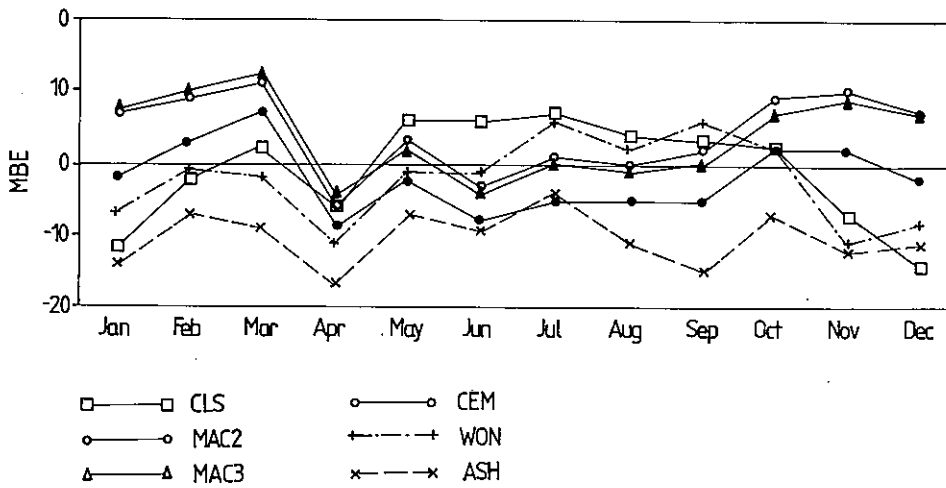


Figure D3.2
Monthly Total MBE (% of Actual) for Global Radiation for 1976 at Goose

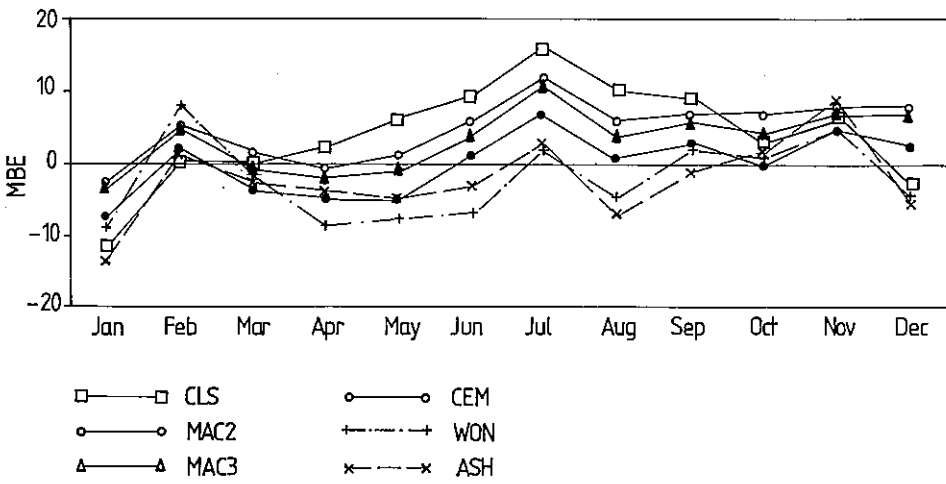


Figure D3.3
Monthly Total MBE (% of Actual) for Global Radiation for 1976 at Toronto

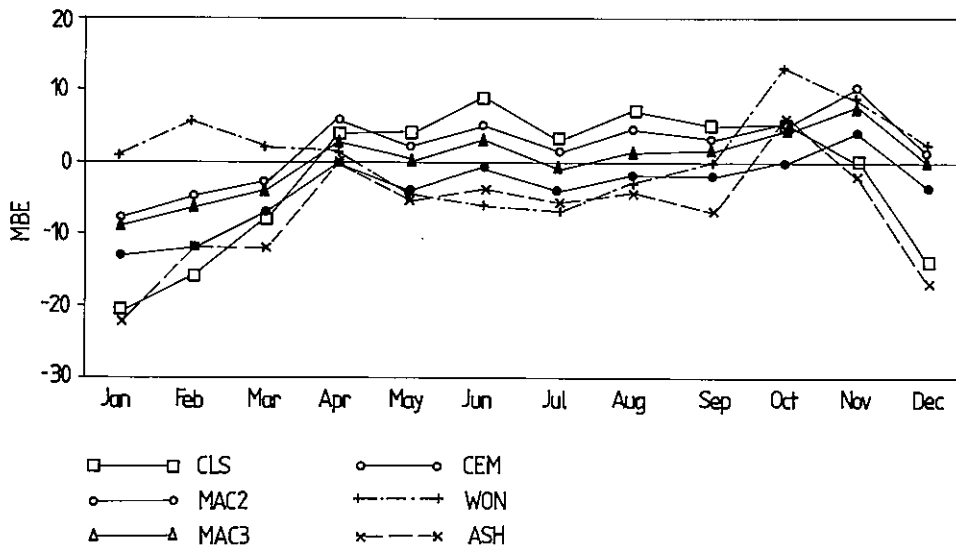


Figure D3.4
Monthly Total MBE (% of Actual) for Global Radiation for 1976 at Winnipeg

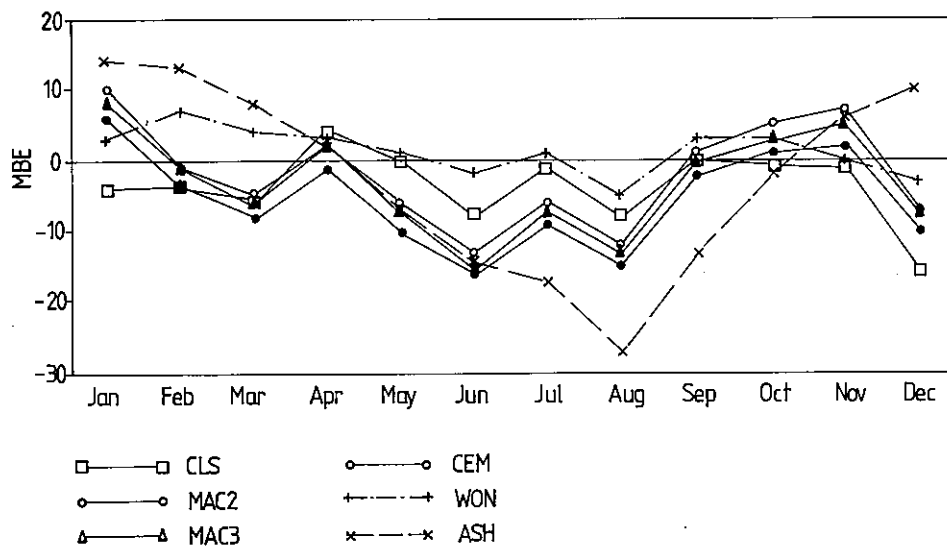


Figure D3.5
 Monthly Total MBE (% of Actual) for Global Radiation for 1976 at Port Hardy

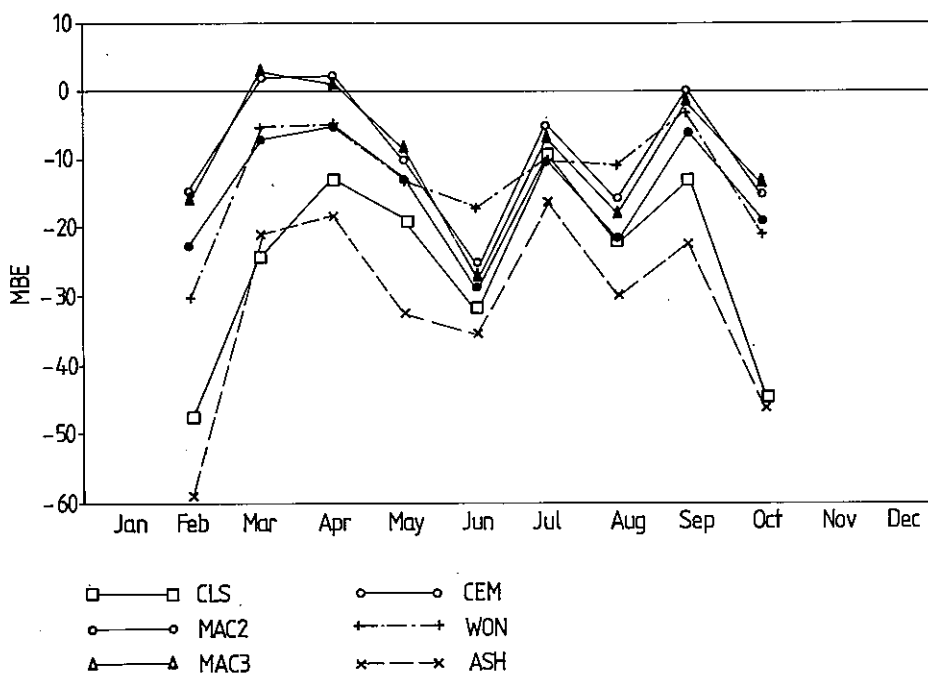


Figure D3.6
 Monthly Total MBE (% of Actual) for Global Radiation at Resolute

A P P E N D I X E (CHAPTER 3)

The following validation examples are contained in the report, Predetermination of Irradiation on Inclined Surfaces for Different European Centres, by Page, (1979). Table E3.1 displays validation results for mean monthly daily totals of global radiation for surfaces of varying slopes and azimuths for Hamburg, Germany using the AVDY-KEW model.

A graphical comparison of two techniques, the AVDY and the Krochmann approach, are illustrated for various surfaces in Figures E3.1, E3.2, E3.3, E3.4 and E3.5. These errors are displayed on a monthly basis for Hamburg, Germany for 1952-54.

Table E3.1
Observed and Predicted Irradiation for Slopes
Hamburg, Germany 1952-54
Using the AVDY - KEW Model and
Morning and Afternoon Sunshine Hours and
Variations in Albedo to
Account for Snow Cover

Mean Error for
Mean Monthly Daily Totals of Global Solar
Radiation ($\text{MJ m}^{-2}\text{d}^{-1}$) for Various Surfaces 1952-54

Month & (Albedo)	45° Tilt South			Vertical South			Vertical East			Vertical West			Vertical North		
	OBS	PRD	(PRD ₋₁)	OBS	PRD	(PRD ₋₁)	OBS	PRD	(PRD ₋₁)	OBS	PRD	(PRD ₋₁)	OBS	PRD	(PRD ₋₁)
	OBS			OBS			OBS			OBS			OBS		
	x 100			x 100			x 100			x 100			x 100		
Jan (.34)	3.31	3.46	+ 4.5	3.52	3.46	- 1.7	1.34	1.47	+ 9.7	1.80	1.49	-17.2	0.80	0.85	+ 7.5
FEB (.45)	5.44	6.08	+11.8	5.44	5.69	+ 4.8	2.89	3.24	+12.1	3.31	3.20	- 3.3	1.76	1.92	+ 9.0
MAR (.24)	9.92	10.80	+ 8.8	9.08	8.75	- 3.6	3.98	5.37	+34.4	5.53	5.70	+ 3.1	2.64	2.98	+12.9
APR (.20)	15.70	16.44	+ 4.7	11.68	11.38	- 2.6	7.42	9.41	+26.8	8.87	9.11	+ 2.7	4.31	4.64	+ 7.7
MAY (.20)	17.33	17.06	- 1.6	11.55	10.46	- 9.4	9.33	10.79	+15.6	10.54	10.28	- 2.5	6.53	6.41	- 1.8
JUN (.20)	16.03	15.77	- 1.6	9.96	9.28	- 6.8	10.09	10.35	+ 2.6	10.38	9.86	- 5.0	7.12	7.07	- 0.7
JUL (.20)	14.11	14.93	+ 5.8	9.71	8.96	- 7.7	9.17	9.00	- 1.9	9.04	9.80	+ 8.4	6.57	6.39	- 2.7
AUG (.20)	13.81	14.02	+ 1.5	9.21	9.14	- 0.8	7.58	7.77	+ 8.1	8.33	8.59	+ 3.1	4.90	4.87	- 0.6
SEP (.20)	12.68	12.64	- 0.3	10.05	9.51	- 5.4	6.45	7.04	+ 9.1	6.78	6.26	- 7.7	3.43	3.44	+ 0.3
OCT (.20)	6.28	5.96	- 5.1	5.53	4.91	-11.2	2.76	2.81	+ 1.8	3.56	3.04	-14.4	1.84	1.76	- 4.4
NOV (.26)	3.73	3.83	+ 2.6	3.56	3.80	+ 6.7	1.42	1.65	+16.2	1.72	1.71	- 0.6	0.92	0.98	+ 6.5
DEC (.24)	1.97	2.26	+14.7	1.93	2.21	+14.5	0.71	0.83	+16.9	1.00	1.00	+ 0.0	0.59	0.56	- 5.1
Annual															
Mean	10.03	10.27	+ 2.4	7.60	7.30	- 4.0	5.26	5.81	+10.5	5.89	5.83	- 1.0	3.45	2.49	+ 1.2
Mean															
Monthly			+ 3.8			- 1.9			+12.7			- 3.0			- 2.1
Error %															
S.D.			5.8			7.4			10.5			7.3			6.3

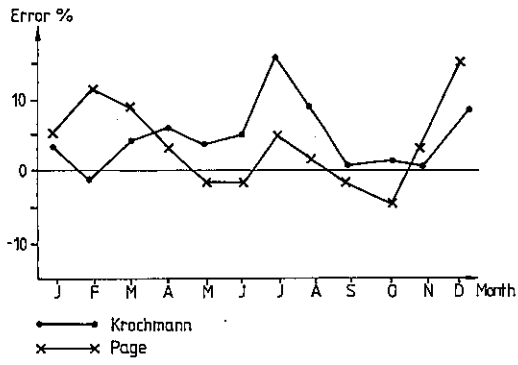


Figure E3.1
Percent Error for Generated Global Radiation
on a South Facing 45° surface

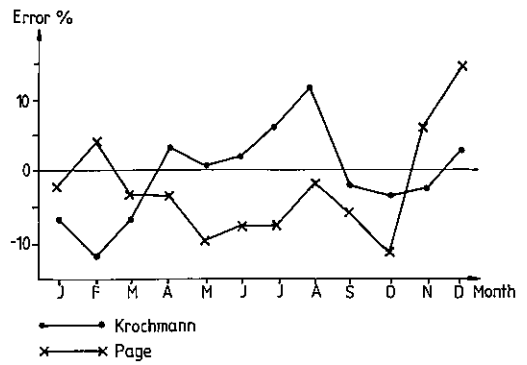


Figure E3.2
Percent Error for Generated Global Radiation
on a South Facing Vertical Surface

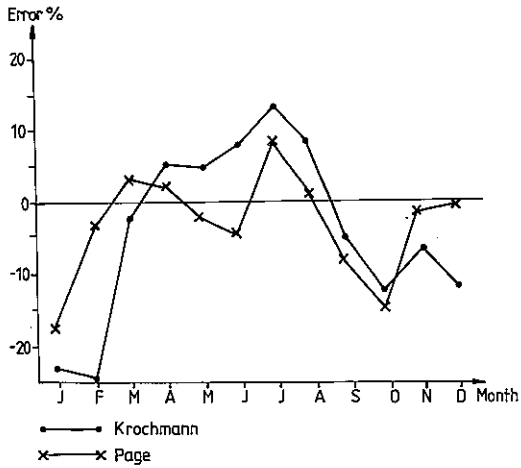


Figure E3.3
Percent Error for Generated Global Radiation
on a West Facing Vertical Surface

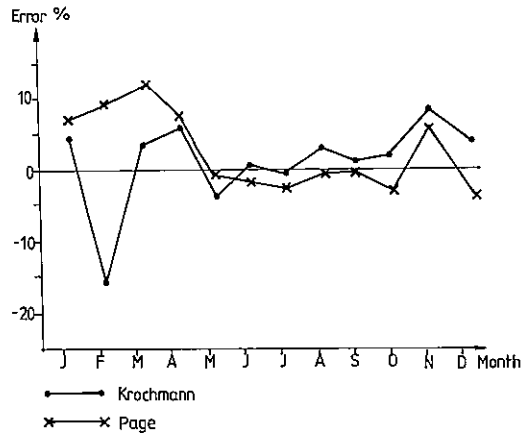


Figure E3.4
Percent Error for Generated Global Radiation
on a North Facing Vertical Surface

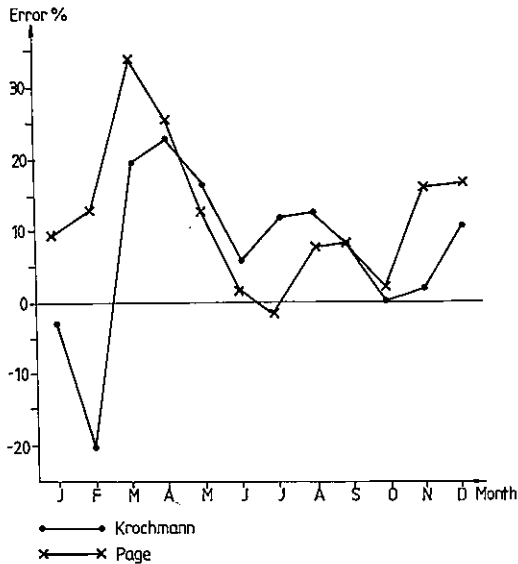


Figure E3.5
Percent Error for Generated Global Radiation
on an East Facing Vertical Surface

APPENDIX F (CHAPTER 4)

Details of Information on Magnetic Tape

Weather data sets for validation of insolation algorithms

1980-09-26

	No. on tape	Sta No.	Radiation				G/ diff/	Sunsh hours	TT	TD	FF	N	CL	Nh	CM	CH	Period
			G	diff	I	GV											
<u>France</u>																	
Odeillo	1	15	X	X	X	S		d								71.04-75.06	
<u>Ireland</u>																	
Valentia	2	28	X	X		S	50°SE	X								76-78.2	
<u>Sweden</u>																	
Norrköping	3	54	X	X	X		60°S 60°S ⁶⁾	X	X	X	X	X	X	X	X	78.07.05- 79-06.27	
<u>U.K.</u>																	
Bracknell	4	70	X	X	3)	N,S,E,W		X								1967-76	
<u>Switzerland</u>																	
Locarno	5	64	X	X		N,S,E,W		X			4)					57.07-71.12	
<u>Italy</u>																	
Ispira	6	31	X	X			60°S		X	rh	X					1979	
<u>Danmark</u>																	
Vaerløse	8	42	X	X		N,S,E,W	60°S	X	X	X	X	8)8)	8)8)	8)8)	8)8)	77-12-78.11	
Vaerløse-Taastrup ⁹⁾	7	43	X	6)	6)	5)		d	X	X	X	8)8)	8)8)	8)8)	8)8)	1959-73	
<u>Belgium</u>																	
Uccle ⁷⁾	9	81	X	X	X			X	X							1958-75	
<u>Comments</u>																	
1)	S	74.07-75-07,	44°S	1973													
2)	48°46S	73-01-74.03,	50°SE	74-03- 74.09.N75.01-75.02													
3)		74.11-76.12															
4)		in tenths, 3 times daily															
5)		only part of period, towards the sun, through a vertical double pane.															
6)		computed values															
7)		half-hourly values in original data															
8)		original value every third hour															
9)		radiation data integrated from .5 h before to .5 h after the hour.															
<u>Abbreviations</u>																	
No.	station identification number on tape																
G	global radiation on horizontal surface																
GV	global radiation on vertical surface																
G/	global radiation on inclined surface																

APPENDIX G (CHAPTER 4)

Record formats etc for the data sets on tape

VIA 1

General Record Formats

DATA	length	place
Station No. without trailing zero	2	1- 2
Year, month, day, hour	4x2	3-10
Indicator	2	11-12
Global radiation	4	13-16
Field for additional data as indicated for each data set		17-

Units

Radiation:	$W \cdot m^{-2}$
Sunshine hours:	per cent
Temperatures:	1/10 deg. Celsius
ff:	$dm \cdot sec^{-1}$
N, Nh:	octas (for Locarno: tenths)
Indicator:	Allways blank unless otherwise indicated particularly for the data set

Missing data

Indicated as -999 or -9 for 4 resp 2 character fields

Data representation

Numerical characters, EBCDIC

Magnetic tape

1600 bpi, 9 tracks, no labels

Particular information on the data sets,

VIA 1

1. Odeillo (original data from CNRS Laboratory)

Additional data	length	place
Diffuse rad.	4	17-20
Direct rad.	4	21-24
CV, south	4	25-28
Not used	12	29-40

Record length: 40
Block length: 3840

2. Valentia

Additional data	length	place
Diffuse rad.	4	17-20
GV, south	4	21-24
Sunshine duration	4	25-28
Not used	4	29-40

Record length: 40
Block length: 3840

3. Norrköping

Additional data	length	place
Diffuse rad.	4	17-20
Direct rad.	4	21-24
G / 60°, south	4	25-28
Diff/60°, south	4	29-32
Sunshine duration	4	33-36
Not used	4	37-40
Air temp.	4	41-44
Dew point temp.	4	45-48
Wind velocity	4	49-52
N	2	53-54
CL	2	55-56
Nh	2	57-58
CM	2	59-60
CH	2	61-62

Record length: 62
Block length: 2976

4. Bracknell

Additional data	length	place
Diffuse rad.	4	17-20
Direct rad.	4	21-24
GV, north	4	25-28
GV, south	4	29-32
GV, east	4	33-36
GV, west	4	37-40
Sunshine duration	4	41-44
Not used	10	45-54

Record length: 54
Block length: 3888

5. Locarno

Additional data	length	place
Diffuse rad.	4	17-20
GV, north	4	21-24
GV, south	4	25-28
GV, east	4	29-32
GV, west	4	33-36
Sunshine duration	4	37-40
Not used	12	41-52
N	2	53-54

Key to indicator value:

- 1: Global radiation value synthetical
- 2: Diffuse radiation value synthetical
- 3: Both said values synthetical
- 0: None of the said values synthetical

Record length: 54
Block length: 3888

6. Ispra

Additional data	length	place
Diffuse rad.	4	17-20
G/60° south	4	21-24
Not used	16	25-40
Air temp.	4	41-44
Relative humidity	4	45-48
Wind velocity	4	49-52
Not used	2	53-54

Record length: 54
Block length: 3888

7. Vaerløse-Taastrup (Copenhagen)

Additional data	length	place
Diffuse rad.	4	17-20
Direct rad.	4	21-24
GV, towards the sun	4	25-28
Not used	12	29-40
Air temp.	4	41-44
Dew point temp.	4	45-48
Wind velocity	4	49-52
N	2	53-54
CL	2	55-56
Nh	2	57-58
CM	2	59-60
CH	2	61-62

The indicator for artificial data etc is an integer value equal to the sum of the code values for all indicated data. The codes of major significance are:

- 1: m.b.t.
- 2: humidity data
- 4: radiation data
- 16: ff, wind velocity
- 32: N, cloud cover
- 64: Nh, cloud cover in low or medium altitude

The value is represented on the magnetic tape as a binary half word. (FORTRAN: Integer *2).

Record length: 62
Block length: 2976

8. Vaerløse

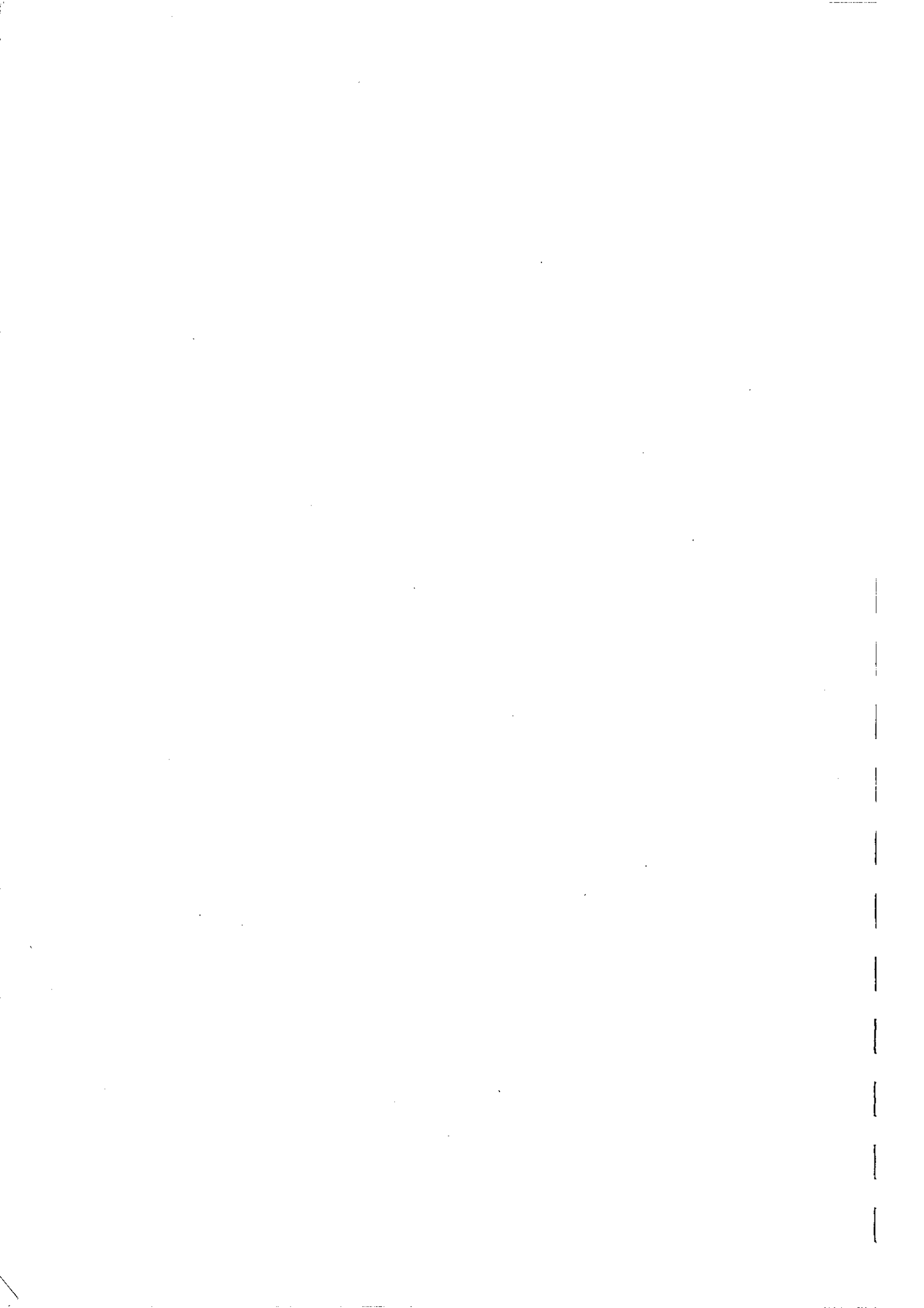
Additional data	length	place
Diffuse rad.	4	17-20
GV, north	4	21-24
GV, south	4	25-28
GV, east	4	29-32
GV, west	4	33-36
Diff/60°, south	4	37-40
Sunshine duration	4	41-44
Air temp.	4	45-48
Wet bulb temp.	4	49-52
Wind velocity	4	53-56
N	2	57-58
CL	2	59-60
Nh	2	61-62
CM	2	63-64
CH	2	65-66

Record length: 66
Block length: 3168

9. Uccle

Additional data	length	place
Diffuse rad.	4	17-20
Direct rad.	4	21-24
Sunshine dur.	4	25-28
Cambell-Stokes sunshine dur.actinom.	4	29-32
Air temp.	4	33-36
Not used	4	37-40

Record length: 40
Block length: 3840



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